

II. Thermodynamics

Thermodynamics, as the most fundamental subject in the field of thermal sciences, is simply defined as “the science that deals with matter, energy, and the laws governing their interactions^{*}”. Thermodynamics plays a vital role in the design and operation of power plants (fossil, nuclear, and solar), direct energy conversion (thermoelectric, thermionic, magnetohydrodynamic, and photovoltaic), heating and cooling systems (boilers, fan coolers, heat pumps, refrigerators, radiators, steam generators, and other heat exchangers), chemical plants (petrochemical refineries, water desalination, air separation, paper production, and pharmaceutical plants), bioengineering systems (lasers, life support systems, artificial heart, CAT scans), and various types of engines (automotive engines, ships, aviation gas turbines, and spacecrafts) among others. Such societal problems as energy shortages, air pollution, and waste management are better understood and remedied through the application of the laws of thermodynamics.

Historically, the development of classical thermodynamics began in late 18th century. In 1760 Joseph Black introduced the concepts of latent heat of fusion and evaporation. He also founded the *caloric theory*. In 1765 James Watt improved his steam engine through the use of an external condenser. However, it was not until the 19th century that the science of thermodynamics flourished. Below we summarize the important milestones in the development of this science in the past 200 years. The new terms and concepts mentioned in this summary are discussed later in this chapter.

- 1816, Robert Stirling patented the first engine using air as the working fluid.
- 1824, Carnot published his work on thermodynamic cycles and the second law of thermodynamics.
- Early 1840s, Julius Robert Mayer and James Prescott Joule introduced theories of the equivalency of heat and mechanical work.
- 1847, Helmholtz formulated the principle of conservation of energy and Emile Clapeyron expanded Carnot’s work
- 1848, William Thomson (Lord Kelvin) defined the absolute temperature scale based on the Carnot cycle.
- 1850, Rudolph Clausius introduced the concept of internal energy, distinguished the specific heat at constant volume from the specific heat at constant pressure, and clarified the distinction between the first and the second laws of thermodynamics.

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- 1859, William Rankine who had been working towards the improvement of practical steam cycles, defined the thermodynamic efficiency of a heat engine, introduced the pressure-specific volume diagram, and published the first thermodynamics textbook.
- 1862, Nikolaus Otto introduced the Otto cycle for reciprocating internal combustion engines.
- 1865, Clausius defined the first law of thermodynamics as “the energy of the universe is constant” and the second law of thermodynamics, “the entropy of the universe tends toward a maximum”.
- 1875, Josiah Willard Gibbs developed the temperature-entropy diagram
- 1878, Gibbs published his work on thermodynamic equilibrium. Gibbs established the field of physical chemistry on the basis of thermodynamics and contributed much towards the establishment of the field of statistical thermodynamics¹.
- 1879, Gottlieb Daimler obtained a patent for a multi-cylinder engine operating on a common crankshaft.
- 1893, Rudolph Diesel introduced the diesel cycle working on the principle of compression stroke to obtain high temperatures for combustion.
- 1897, Max Planck stated the second law of thermodynamics.
- 1899, Karl Benz improved Daimler’s engine by introducing the controlled-timing electric ignition system.

In this book, the topic of thermodynamics is divided into three chapters. This chapter deals with the fundamentals of thermodynamics, the second chapter (IIb) discusses thermodynamic cycles for power production, and in the third chapter (IIc) the application of mixtures of non-reacting ideal gases is discussed.

Ila. Fundamentals

In the present chapter dealing with the fundamentals of thermodynamics, we introduce such basic concepts as a system and its surroundings, system properties, system processes, and possible direction of a process. We also explore the effect of the flow of mass and energy (in the form of heat and work) on a system. However, we first need to present the definition of these and other important terminologies as related to a thermodynamic substance and its related state. Subsequent to the definition of terms, we introduce the equations of state for two widely used working fluids: air and water. We then proceed with the definition of the three laws of thermodynamics. Examples are provided for both steady state and transient conditions using the conservation equations of mass and energy. Since these equations are dealt with in Chapter III, we use them in this chapter without further derivation.

¹ J. Willard Gibbs is also the founder of *Vector Analysis* (see Chapter VIIc).

1. Definition of Terms

1.1. Definitions Pertinent to Dimensions and Units

Dimensions are names applied to such physical quantities as length (L), mass (m), time (t), and temperature (T). These dimensions are known as the *primary dimensions**. We may also include the electric current (q) and the luminous intensity (I) as primary dimensions. All other physical quantities can be expressed in terms of these primary or fundamental dimensions. For example, velocity $V = x/t$, can be expressed as $[V] = Lt^{-1}$, density $\rho = m/V$ as $[\rho] = mL^{-3}$, force $F = ma$ as $[F] = mLt^{-2}$, pressure $P = F/A$ as $[P] = mL^{-1}t^{-2}$, etc. Symbols are placed inside brackets to signify the dimension of a physical quantity. It is of prime importance to ensure the dimensional validity of engineering formulae derived from the first principles. Hence, the dimensions of both sides of an equation must match. This is known as the *principle of dimensional homogeneity*.

Units are measures of a dimension and depend on the standard used for the unit system. There are two unit systems in use, the SI (short for its French expression, *Le Systeme Internationale d'Unites*) system of units and the English engineering system of units, referred to in this book as British Units, or BU for short. In the table below, units of the primary dimensions are expressed in both SI and BU. Other units can be derived from the basic units for such physical quantities as force, pressure, energy, power, etc. Force, for example has a derived unit that, according to Newton's second law of motion, is related to mass and acceleration so that $F \propto ma$. The derived unit for force in the SI system of units is $\text{kg}\cdot\text{m}/\text{s}^2$. In this system, force is expressed in Newton (N). Hence, one Newton, is the amount of force that would accelerate a mass of 1 kg at a rate of $1 \text{ m}/\text{s}^2$. If we now introduce a proportionality factor shown by g_c , then Newton's second law can be written as:

$$F = ma/g_c$$

Physical Quantity	Basic SI Unit	SI Symbol	Basic BU	BU Symbol
Length	Meter	m	Foot	ft
Mass	Kilogram	kg	Pound	lbm
Time	Second	s	Second	s
Temperature	Degree Kelvin	K	Degree Rankine	R

It is clear that g_c has a value of unity and units of $[g_c] = \text{kg}\cdot\text{m}/(\text{N}\cdot\text{s}^2)$. Force may also be expressed in terms of kilogram force, (kgf) which is the amount of force that would accelerate a mass of 1 kg at a rate of $9.8 \text{ m}/\text{s}^2$. To prevent confusion, the symbol for mass is also shown as kgm, which stands for Kilogram mass.

In British Units, the symbol of force is pound force (lbf), accelerating a mass of 1 slug at a rate of $1 \text{ ft}/\text{s}^2$. The most frequently used unit for mass is pound mass

* Also see Appendix I where physical quantities are expressed in both mass, length, and time (*MLT*) and force, length, and time (*FLT*) systems.

(lbm), which is accelerated at a rate of 32.1740 ft/s^2 by a force having the magnitude of one pound force. In British Units, g_c has a value of 32.174 (usually used as 32.2) and units of $[g_c] = \text{lbm} \cdot \text{ft} / \text{lbf} \cdot \text{s}^2$.

$$g_c = 1 \text{ kg} \cdot \text{m} / \text{N} \cdot \text{s}^2 \quad g_c = 32.2 \text{ lbm} \cdot \text{ft} / \text{lbf} \cdot \text{s}^2 \quad g_c = 1 \text{ slug} \cdot \text{ft} / \text{lbf} \cdot \text{s}^2$$

Derived units of some physical quantities are shown below.

Physical Quantity	Basic SI Unit	SI Symbol	Basic BU	BU Symbol
Force	Newton	N	Pound Force	lbf
Pressure	Pascal (N/m^2)	Pa	Pressure (lbf/in^2)	psi
Energy	Joule ($\text{N} \cdot \text{m}$)	J	Btu*	Btu
Power	Watt (J/s)	W	Btu/h	Btu/h

* Btu stands for British thermal unit

Example IIa.1.1. A substance has a mass equal to 2 lbm. Find the weight of this substance on the earth's surface and on a planet having $g_{\text{planet}} = \frac{1}{2} g_{\text{Earth}}$.

Solution: By definition, if a substance having mass m is exposed to the gravitational acceleration g , the resulting force is the weight of the mass given by $W = mg/g_c$. On earth, $g_{\text{Earth}} = 32.2 \text{ ft/s}^2$ hence, $W_{\text{Earth}} = 2 \times (32.2/32.2) = 2 \text{ lbf}$. On a planet with $g_{\text{planet}} = 32.2/2 = 16.1 \text{ ft/s}^2$, the weight of the substance is: $W_{\text{planet}} = 2 \times (16.1/32.2) = 1 \text{ lbf}$.

In British units, work is the result of force (lbf), multiplied by distance (ft), to obtain units of $\text{ft} \cdot \text{lbf}$. Heat, on the other hand, is generally expressed in terms of Btu. The conversion factor from $\text{ft} \cdot \text{lbf}$ to Btu is given as $1 \text{ Btu} = 778.16 \text{ ft} \cdot \text{lbf}$ (usually used as 778).

1.2. Definitions Pertinent to a Substance

Pure substance is a homogeneous substance with the same chemical composition in various phases, as defined below. Water, for example, is a pure substance as it has the same chemical composition whether in the form of steam, ice, or liquid water. Air, on the other hand, being a mixture of various gases, is not a pure substance, as in very low temperatures various components would condense at different temperatures resulting in a different chemical composition in the liquid phase.

Phase is a quantity of a pure substance that is homogeneous throughout the substance and is in the form of solid, liquid, or vapor.

Fluid is a term applied to either the liquid or the vapor phase of a pure substance. The *working fluid* is any fluid for which we are studying the thermodynamic behavior during a transformation.

System in a thermodynamic sense, refers to an entity being studied. This entity may be a pure substance such as a lump of matter, a small cylinder containing a mixture of gases, a large pipeline, or an entire power plant. The entity does not

necessarily require possessing any matter. Thus, vacuum may also constitute a thermodynamic system. The role of a system in thermodynamics is similar to the role of a free body diagram in solid mechanics, which is used to study the forces acting on a body. More specific definition for a system is given in Section 4 of this chapter.

Surroundings is anything external to a thermodynamic system. For example, if a system consists of a gas contained in a container, the rest of the universe is considered the surroundings for this system.

Boundary separates a system from its surroundings. The boundary is also known as the *control surface*.

Property such as color, pressure, temperature, density, energy, etc. is an observable characteristic of a system. As is discussed in Section 4, *heat* and *work* are not properties of a system.

Equilibrium is an important concept in thermodynamics. Systems in equilibrium do not experience any change with time. There are several types of equilibrium including thermal, mechanical, chemical, internal, and external. By thermal equilibrium we mean the temperature is the same throughout the system and is equal to the temperature of the surroundings, (which is everything external to the system). By mechanical equilibrium we mean that a system has no unbalanced force within it and the force it exerts on its boundary is balanced by an external force. By chemical equilibrium we mean that the chemical composition of a system remains unchanged. Internal equilibrium occurs in isolated systems and external equilibrium applies to systems that are in an internal equilibrium state and are also in equilibrium with their surroundings. A system is in equilibrium if pressure, temperature, and density are uniform throughout the system and do not change with time.

State. Properties of a system define the state of a system when the system is at equilibrium. For example, if certain amount of a gas at equilibrium is kept in a cylinder equipped with a piston, at pressure P_1 and temperature T_1 we refer to this condition as state 1. If, for whatever reason, the pressure and temperature of the gas are changed to P_2 and T_2 , then the state of the system is changed to state 2. Thus the state of a system changes when properties of a system change. Any change in the state is a deviation from equilibrium.

Gas, vapor, and steam. Gas is a state of matter having low density, low viscosity, high expansion, and compression ability in response to relatively small changes in pressure and temperature. Gases diffuse readily and have the ability to distribute uniformly throughout a system. Vapor is the gaseous state existing below the critical temperature (as defined in Section 1.5) of a substance that is liquid or solid in normal conditions. Steam is a special term applied only to the vapor phase of water.

1.3. Definitions Pertinent to Thermodynamic Processes

Process is applied to any transformation of a system between two equilibrium states; for the cylinder containing gas at state 1 (P_1 , T_1 , and volume V_1), compression of this gas by a piston to bring the gas to pressure P_2 , temperature T_2 , and volume V_2 is referred to as a process. A process is also known as a **Path**.

Isobaric is a process that takes place at constant pressure, such as boiling water in an open container.

Isothermal is a process that takes place at constant temperature. For example, steam condensation on the cold walls of a sauna.

Isochoric (also known as **isometric**) is a process that takes place at constant volume, such as heating a gas in a sealed rigid vessel. Other processes, such as *adiabatic*, *isentropic*, and *polytropic*, are defined in Section 4.

1.4. Definitions Pertinent to Properties of a Substance

We assume that fluid properties vary continuously (see definition of *continuum* in Chapter IIIa).

Pressure is the normal component of force per unit area exerted by a system on its boundary. The most often used pressure is referred to as absolute pressure. Zero pressure and atmospheric pressure (also known as Barometric pressure) are used as reference for absolute pressure (psia in BU). If absolute pressure is measured with respect to the atmospheric pressure, it is referred to as gage pressure (psig in BU). These are clarified in Figure IIa.1.1(a).

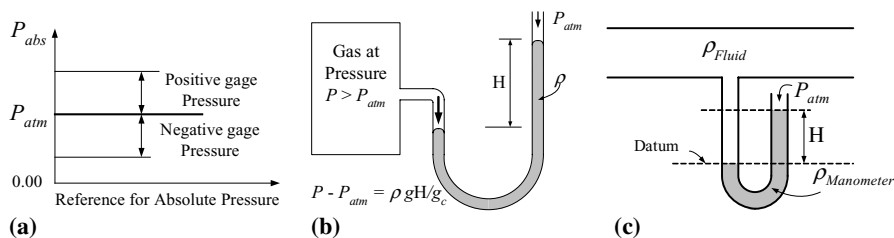


Figure IIa.1.1. (a) Absolute versus gage pressure; (b) manometer; (c) manometer with comparable densities

Manometric pressure is the pressure measured by a manometer, as shown in Figure IIa.1.1(b). In this figure ρ is the density of the manometer liquid, g in the gravitational acceleration, H is the difference in the liquid levels, and g_c is a conversion factor as described in Section 1.1. In Figure IIa.1.1(b), the density of the gas is considered negligible compared with the density of the manometer liquid.

Example IIa.1.2. Find the pressure, in N/m^2 (Pa) and in psia, at the depth of 50 m below the surface of a lake. Note that $1 \text{ atm} = 10.4 \text{ m-H}_2\text{O}$ ($\rho_{\text{Water}} = 1000 \text{ kg/m}^3$) = $14.7 \text{ psi} = 33.92 \text{ ft-H}_2\text{O} = 101.3 \text{ kPa}$.

Solution: At the depth of 50 m, the absolute pressure is $10.4 + 50 = 60.4 \text{ m H}_2\text{O}$. This is equivalent to: $P = \rho (g/g_c)h = 1000(9.8/1) \times 60.4 = 0.595\text{E}6 \text{ N/m}^2 \cong 0.595 \text{ MPa} = 0.5965\text{E}6/6,895 \cong 86 \text{ psia}$.

If the density of the fluid is comparable with the density of the manometer liquid, as shown in Figure IIa.1.1(c) then the fluid density needs to be accounted for. To find the fluid pressure, we perform a force balance with respect to the datum. Since surface areas are equal, they cancel out. The force on the left leg of the manometer is represented by $P_{\text{Fluid}} + \rho_{\text{Fluid}}gH$. This force is balanced by the force exerted in the right leg of the manometer represented as $P_{\text{atm}} + \rho_{\text{Manometer}}gH$. In equilibrium, these forces must be equal, hence:

$$P_{\text{Fluid}} = P_{\text{atm}} + (\rho_{\text{Manometer}} - \rho_{\text{Fluid}})gH$$

Shown in Figure IIa.1.2 are various units for atmospheric pressure.

Fehler! Keine gültige Verknüpfung.

Standard Temperature & Pressure (STP)		
System	Temperature	Pressure
SI	273.15 K	101.325 kPa
Scientific	0.0 C	760 mm Hg
Natural gas	60 F	14.7 psia
Engineering	32 F	14.696 psia

(a)

(b)

Figure IIa.1.2. (a) Various units for atmospheric pressure; (b) commonly used values for standard P & T

Vapor pressure. For a given temperature, every liquid has a vapor pressure at which liquid begins to boil and is at equilibrium with its own vapor. If the liquid is at a pressure greater than its own vapor pressure then there is only evaporation at the interface between the liquid and its vapor. If the liquid pressure drops below its vapor pressure, bubbles form in the liquid. Water at 102 F (39 C) has a vapor pressure of 1 psia ($\sim 7 \text{ kPa}$). Similarly, water at 212 F (100 C) has a vapor pressure of 14.696 psia (101.3 kPa). Values of the standard atmospheric pressure in various units are shown in Figure IIa.1.2(a).

Temperature is a measure of coldness or hotness of a body and is expressed in either Fahrenheit (F) or Celsius (C). Both of these temperature scales are based on

the freezing (0 C and 32 F) and boiling points of water (100 C and 212 F at atmospheric pressure). In the Celsius scale, the range between the freezing and the melting point of water is divided into 100 units. The Fahrenheit and the Celsius scales are related as

$$\frac{F - 32}{180} = \frac{C}{100}$$

Since both of these temperature scales allow for negative temperatures, an absolute temperature that has only positive values is defined. Kelvin (K) and Rankine (R) are the absolute scales for the Celsius and the Fahrenheit temperature scales, respectively. The relations are:

$$K = C + 273.15 \text{ (Generally, 273 is used in practice)}$$

$$R = F + 459.67 \text{ (Generally, 460 is used in practice)}$$

Standard condition refers to a temperature of 0 C (273 K) and atmospheric pressure (760 mm of mercury, Hg). Since volume and density of gases are sensitive to relatively small changes in temperature and pressure, it is customary to reduce all gas volumes to standard conditions for purpose of comparison. Values for standard atmosphere pressure and temperature (STP) are shown in Table IIA.1.2(b).

Specific volume of a substance is the inverse of the density of that substance. Hence, specific volume ($v = V/m$) is the volume per unit mass of a substance and is expressed in units of ft³/lbm or m³/kg. Pressure (P), temperature (T), and specific volume (v) are properties of a substance. As discussed in Section 2, the property surface of a substance is constructed based on P - T - v .

Specific gravity (S_g) of a liquid is the ratio of the density of the liquid to the density of water (62.4 lbm/ft³ or 1000 kg/m³). Specific gravity of a gas is the ratio of the molecular weight of a gas to the molecular weight of air (28.97).

Specific heat is the amount of heat required to raise the temperature of a unit mass of a homogenous phase of a substance by one degree. If the process takes place with either volume or pressure kept constant, the term is respectively referred to as constant-volume (c_v) or constant-pressure (c_p) specific heat, as defined later in this section. Specific heat has units of kJ/kg·K or Btu/lbm·F.

Energy is the ability to lift a weight to a higher elevation. The term, *energy*, is in fact a combination of two Greek words meaning *capacity* and *work*. Energy is a system quantity that describes the thermodynamic state of the system. Energy may be transferred to or from the system. Energy is generally expressed in such units as Joule (J), kilojoules (kJ), British thermal unit (Btu), or the less frequently used foot-pound force (ft·lbf).

Kinetic energy (KE) of a system is the energy associated with the motion of a system relative to a frame of reference, which is usually the earth's surface². Con-

² The relation between kinetic energy and temperature of subatomic particles is discussed in Chapter VIe

sider a body of mass m initially at rest. If an applied force F acts to accelerate mass m , then according to Newton's second law $F = ma$ where a is the resulting acceleration and g_c is implicitly accounted for. If the applied force causes the body to move by dx in the direction of the applied force, then $Fdx = madx$. Substituting for both distance and acceleration in terms of velocity, $dx = Vdt$ and $a = dV/dt$ then $Fdx = mVdV$ where V is an average velocity. Integrating the resulting equation we find³:

$$KE = \int_0^L Fdx = m \int_0^V VdV = mV^2/2$$

Potential energy (PE) of a system is the energy associated with the position or configuration of the system in a potential field such as a gravitational or electromagnetic field. Consider mass m located at height Z with respect to a reference in a gravitational field having a gravitational acceleration g . Mass m then possesses a potential energy given as $PE = mgZ$. Units for potential and kinetic energies are the same as the units for energy.

Total energy (E) of a system is the summation of all the energies possessed by the system including potential, kinetic, and internal energies.

Internal energy (U) of a system is the total energy of the system minus the potential and the kinetic energies, $U = E - (KE + PE)$. The internal energy represents the energy on the microscopic level. As described by Reynolds, it consists of such energies as nuclear and molecular binding energies, molecular rotation, translation, and vibration, intermolecular weak and strong energies, mass equivalent energy, and such other microscopic energies associated with the nuclear and electron spin.

Enthalpy (H) of a system is defined as the summation of the internal energy (U) and pressure work (PV), as in $H = U + PV$. Enthalpy and internal energy have units of J or Btu. To avoid errors associated with unit conversion, we may write $H = U + cPV$ where $c = 1$ for H and U expressed in J, P in Pa, and V in m^3 . The value of c in British Units is $c = 144/778 = 0.185$ for U and H expressed in Btu, P in psia, and V in ft^3 .

Entropy (S) is a measure of the disorder of a system. The change in the entropy of a system is always greater, or at least equal, to the heat transfer to or from the system divided by the temperature of the system. Entropy has the units of J/K or Btu/R. Specific entropy ($s = S/m$) has units of kJ/kg·K or Btu/lbm·R = 4.1868 kJ/kg·K.

Specific heat (c_v). The specific heat of a substance at constant specific volume is defined as $c_v = (\partial u / \partial T)_v$, where u , the specific internal energy, is given as $u = U/m$.

³ The rotational kinetic energy is $KE = I\omega^2/2$ where I is the moment of inertia and ω is the angular velocity given as $\omega = 2\pi N$ with N representing revolution per second.

Specific heat (c_p). The specific heat of a substance at constant pressure is defined as $c_p = (\partial h / \partial T)_P$, where h , the specific enthalpy, is given as $h = H/m$.

1.5. Definitions Pertinent to Types of Properties

Thermodynamic properties are such quantities as pressure (P), density (ρ), temperature (T), enthalpy (H), entropy (S), specific heat (c_p and c_v), coefficient of thermal expansion (β), and bulk modulus (B).

Transport properties refer to such quantities as viscosity (μ) and turbulent diffusivity (\mathcal{E}) as discussed in Chapter IIIa, and thermal conductivity (k) as discussed in Chapter IVa.

Extensive and intensive properties are defined to distinguish properties that depend on the size of the system (extensive) from those that do not depend on the size of the system (intensive). Such system properties as volume (V), mass (m), momentum (mV), enthalpy (H), and entropy (S) are examples of extensive properties. Examples of intensive properties include temperature (T), pressure (P), density ($\rho = m/V$), specific volume ($v = V/m$), specific enthalpy ($h = H/m$) and specific entropy ($s = S/m$). These are summarized in Table IIa.1.1. The state of a substance is determined by two intensive properties. These could be pressure and temperature, pressure and specific internal energy, temperature and specific enthalpy, etc.

Table IIa.1.1. Examples of extensive and intensive properties

Property	Extensive (Y)	Intensive (y)
Mass	m	ρ
Volume	V	v
Momentum	mV	V
Kinetic energy	$\frac{1}{2} mV^2$	$\frac{1}{2} V^2$
Potential energy	mgZ	gZ
Internal energy	U	u
Total energy	E	e
Enthalpy	H	h
Entropy	S	s

Critical state of a substance is a state beyond which a liquid-vapor transformation is not possible. For water, the critical pressure is $P_c = 22$ MPa (3203.6 psia), and critical temperature is $T_c = 374.15$ C (705.47 F).

Critical properties. Pressure, temperature, and specific volume (P_c , T_c , v_c) of the critical state are referred to as critical properties. Using P_c and T_c for water, the critical volume for water is given as $v_c = 0.0505$ ft³/lbm.

Reduced properties refers to the ratios of pressure and temperature normalized to corresponding critical pressure and temperature, respectively ($P_R = P/P_c$ and $T_R = T/T_c$).

2. Equation of State for Ideal Gases

The state of a substance is a function of two independent intensive properties. Mathematically, a function of two variables represents a surface in rectangular coordinates. The functional relationship between various properties of a substance in terms of the two independent intensive properties is referred to as the equation of state. For example, if we choose the two independent intensive properties as pressure (P) and temperature (T) a relation that expresses specific volume in terms of these properties, such as $v = f(P, T)$, is an equation of state with P and T being the independent variables. In this section, the equation of state for ideal gases is discussed following the definition of some pertinent terms.

2.1. Definition of Terms

Atomic mass of elements is measured with respect to the mass of Carbon 12. We define an atomic mass unit as $1/12^{\text{th}}$ of the mass of the atom of C_6^{12} . This minute amount of mass is equal to $1.660438 \text{ E-}27 \text{ kg}$. Hence, the atomic mass of an element is the mass of an atom on a scale that assigns C_{12} a mass of exactly 12.

Molecular weight of a compound is the sum of the atomic weights of the atoms that constitute a molecule of the compound.

Gram-mole. A gram-mole (mol in the SI system) of a substance is the amount of that species whose mass in gram is numerically equal to its molecular weight. For example, carbon monoxide (CO) has a molecular weight of 12 (for Carbon) + 16 (for Oxygen) = 28. In general, if the molecular weight of a substance is M , then there are $M \text{ kg/kmol}$ or $M \text{ lbm/lb-mol}$ of this substance.

Example IIa.2.1. Find the number of moles in 80 kg of CO_2 .

Solutio: Since $M_{\text{CO}_2} = 12 + 2 \times 16 = 44$, then the number of CO_2 moles are $80/44 = 1.82 \text{ kmol CO}_2$.

2.2. Equation of State

All gases at sufficiently low pressures and high temperatures (hence, at low density) obey three rules: Boyle's, Charles', and Gay-Lussac's rules. These are called the *perfect gas* rules and such gases are known as *perfect* or *ideal* gases. While the perfect gas and ideal gas are used interchangeably, an ideal gas is a perfect gas with an additional feature of having constant specific heat. *Boyle's rule* specifies that in isothermal processes, $PV = \text{constant}$. *Charles' rule* specifies that in an iso-

baric process, $V/T = \text{constant}$. Finally, *Gay-Lussac's rule* specifies that in constant volume processes, $P/T = \text{constant}$.

From any two of the above rules we conclude that for a given mass of an ideal gas, $PV/T = \text{constant}$. In order to determine the constant, we take advantage of the *Avogadro's hypothesis*, which states that, at the same pressure and temperature, equal volumes of gases contain the same number of molecules. In other words, 22.4 liters of any gas at STP, contains 1 mole or 6.023×10^{23} molecules of that gas. This is known as the *molar volume*. From the Avogadro's hypothesis we may conclude that the constant, shown by R_u , is given as $R_u = P\bar{v}/T = (1 \text{ atm} \times 22.4 \text{ liter})/(1 \text{ mole} \times 273 \text{ K}) = 0.0821 \text{ atm} \cdot \text{liter} \cdot \text{K}^{-1} \cdot \text{mole}^{-1}$. In this relation, T is the absolute temperature, \bar{v} is specific volume on a molar basis, and R_u is known as the *universal gas constant* and its value can also be found in such units as:

$$\begin{aligned} R_u &= 8.314 \text{ kJ} \cdot \text{kmol}^{-1} \cdot \text{K}^{-1} & R_u &= 0.08314 \text{ bar} \cdot \text{m}^3 \cdot \text{kmol}^{-1} \cdot \text{K}^{-1} \\ R_u &= 1545 \text{ ft} \cdot \text{lbf} \cdot \text{R}^{-1} \cdot \text{lbmol}^{-1} & R_u &= 0.73 \text{ atm} \cdot \text{ft}^3 \cdot \text{lbmol}^{-1} \cdot \text{R}^{-1} \end{aligned}$$

The equation of state for an ideal gas, $P\bar{v} = R_u T$ can be written as $Pv = RT$ where $v = \bar{v}/M$ and $R = R_u/M$. Alternatively, we can write:

$$PV = nR_u T = m(R_u/M)T = mRT \quad \text{IIa.2.1}$$

where m is mass (kgm or lbm), M is molecular weight (kg/kgmol or lb/lbmol), n is the number of moles, and R is given as $R = R_u/M$ ($\text{kPa} \cdot \text{m}^3/\text{K} \cdot \text{kg}$ or $\text{ft} \cdot \text{lbf}/\text{R} \cdot \text{lbm}$). Note that unlike R_u , which is a universal constant, the value of R depends on a specific ideal gas. Also note that in Equation IIa.2.1, we made the following substitution:

$$n = m/M$$

That is to say that one mole (mol) of any substance has a mass equal to its molecular weight.

Example IIa.2.2. A 10 ft^3 (0.283 m^3) tank contains compressed air at a pressure of 350 psia (2.41 MPa) and temperature of 80 F (27 C). We want to determine the mass of air and moles of air in this tank. $M_{\text{air}} = 28.97$.

Solution: From the equation of state for ideal gases using the air molecular weight of 28.97 lb/lbmole:

$$\begin{aligned} m &= \frac{PV}{(R_u/M)T} = \frac{\text{BU} \quad (350 \times 144) \times 10}{(1545/28.97) \times (80 + 460)} = 17.5 \text{ lbm} = \\ &\quad \text{SI} \quad \frac{(2.41 \text{E}3) \times 0.283}{(8.314/28.97) \times (27 + 273)} = 7.92 \text{ kg} \end{aligned}$$

The number of moles is found from $n = 17.5/28.97 = 0.6 \text{ lbmol}$ or alternatively, $n = 7.92/28.97 = 0.27 \text{ kmol}$.

The advantage of the equation of state for an ideal gas is its simplicity. Although in texts on thermodynamics the Boyle, Charles, and Gay-Lussac rules are generally referred to as “laws”, they were introduced here as “rules” because their application is limited only to gases that can be approximated as ideal gas. We can approximate the behavior of real gases with that of an ideal gas only if the compressibility of the gas is near unity. The compressibility of a gas is defined as $Z = Pv/RT$. When $Z \approx 1$, the gas density is low enough to allow the treatment of the gas as an ideal gas. There have been several attempts to develop an equation of state for non-ideal or real gases ($Z \neq 1$). For example, Van der Waals in the 19th century proposed the following equation of state:

$$(P + \frac{c_1}{v^2})(v - c_2) = RT \quad \text{IIa.2.2}$$

where, c_1 and c_2 are functions of P_c and T_c . Note the Van der Waals equation reduces to the ideal gas equation for large values of specific volume (occurring at low pressures or high temperatures).

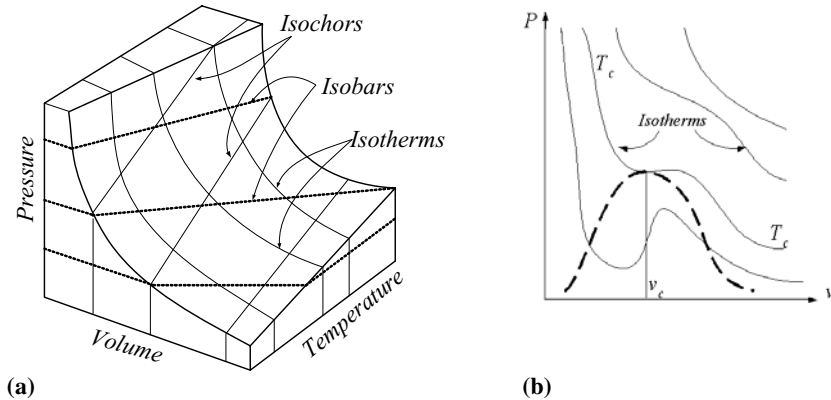


Figure IIa.2.1. Behavior of (a) an ideal gas and (b) a Van der Waals gas

To find the values of c_1 and c_2 in Equation IIa.2.2, we note that the isotherm passes through a point of inflection at the critical state hence, the first and the second derivatives of pressure with respect to specific volume at constant temperature are zero. These in addition to Equation IIa.2.2, provide three equations from which we can find $c_1 = 0.42R^2T_c^2 / P_c$, $c_2 = 0.125RT_c / P_c$ and $v_c = 0.375RT_c / P_c$. Having v_c , we can find Z from $Z = P_c v_c / (RT_c) = 0.375$.

The functional relationship between P , T , and v for the ideal gas and the Van der Waals gas are shown in Figure IIa.2.1. The Van der Waals equation, while an improvement over the ideal gas model, has limited applications. To correlate pressure, specific volume, and temperature, there have been many other equations of states since the introduction of the Van der Waals equation. Among these are Berthelot, Dieterici, and Redlich-Kwong (RK) equations. The Redlich-Kwong equation for example is an empirical correlation and is given as:

$$P = \frac{R_u T}{\bar{v} - c_2} - \frac{c_1}{\bar{v}(\bar{v} + c_2)T^{0.5}} \quad \text{IIa.2.3}$$

where $\bar{v} = Mv$ has units of ft^3/lbmol or m^3/kgmol . Constants c_1 , and c_2 in Equation IIa.2.3 are functions of the critical pressure and temperature and are given as $c_1 = 0.4275 R_u^2 T_c^{2.5} / P_c$ and $c_2 = 0.0867 R_u T_c / P_c$.

Example IIa.2.3. Use the Redlich-Kwong equation and find the pressure of superheated vapor given a specific volume of $2.7247 \text{ ft}^3/\text{lbm}$ ($0.17 \text{ m}^3/\text{kg}$) at 500 F (260 C).

Solution: For water we have $P_c = 3203.6 \text{ psia}$ and $T_c = 705 \text{ F}$. Therefore,
 $c_1 = 0.4275 [1545/(144 \times 14.7)]^2 (705 + 460)^{2.5} / (3203.6/14.7) = 48,422 \text{ atm}$
 $(\text{ft}^3/\text{lbmol})^2 \text{ R}^{0.5}$
 $c_2 = 0.0867 [1545/(144 \times 14.7)] (705 + 460) / (3203.6/14.7) = 0.338 \text{ ft}^3/\text{lbmol}$.
 $T = 500 + 460 = 960 \text{ R}$, $\bar{v} = 18 \times 2.7247 = 49.0446 \text{ ft}^3/\text{lbmol}$. Substituting in Equation IIa.2.3, we get:

$$P_{RK} = \frac{[1545/(14.7 \times 144)]960}{(49.0446 - 0.338)} - \frac{48,422}{49.0446(49.0446 + 0.338)\sqrt{960}} =$$

$$13.74 \text{ atm} = 201.9 \text{ psia} (1.39 \text{ MPa})$$

From the ideal gas model, we find $P_{IG} = [(1545/18) \times 960] / 2.7247 = 30242 \text{ lbf/ft}^2 = 210 \text{ psia}$ (1.44 MPa). The real answer is 200 psia . In this example, we do not expect to get good results from the ideal gas model, as pressure is not low enough and temperature is not high enough (try $P = 1 \text{ psia}$ and $T = 750 \text{ F}$).

It is seen from the above example that while the Redlich-Kwong model does a better job in predicting pressure, it still has an error of about 1%. To get even closer answers, more complex equations should be used. These include the Beattie-Bridgeman and the Benedict-Webb-Rubin equations.

2.3. Specific Heat of Ideal Gases

Joule showed that for ideal gases, the internal energy is only a function of temperature, $u = u(T)$. As such, for an ideal gas the partial derivative becomes a total derivative hence, we can write $du = c_v dT$. Similarly, for infinitesimal changes in enthalpy $dh = c_p dT$. By definition, enthalpy is related to internal energy as $dh = du + Pdv$. This relation can be applied to an ideal gas by substituting for the last term in the right-hand side from the equation of state, to get $dh = du + RdT$. Substituting for du and dh in terms of specific heats for an ideal gas yields:

$$c_p - c_v = R$$

Since R is constant and for an ideal gas c_v is only a function of temperature, c_p also becomes only a function of temperature. The specific heat ratio is defined as $\gamma = c_p/c_v$. Combining these two equations, we can solve for c_v and c_p in terms of R and γ :

$$c_v = \frac{R}{\gamma - 1}, \quad c_p = \frac{\gamma R}{\gamma - 1}$$

Note that c_v and c_p have the same units as R . These are kJ/kg·K, in SI or Btu/lbm·R, in British units.

Example IIa.2.4. Calculate c_v and γ of an ideal gas which has a molar mass of 16 and a $c_p = 2$ kJ/kg·K.

Solution: We first calculate R from $R = R_u/M$. Hence, $R = 8.314/16 = 0.519$ kJ/(kg·K). Having R and c_p , we can find $c_v = c_p - R$. Substituting, $c_v = 2 - 0.519 = 1.48$ kJ/kg·K. Having c_p and c_v , we find $\gamma = 2/1.48 = 1.35$.

Specific heat of ideal gases at constant pressure may be expressed in the form of a quadratic polynomial. For all practical purposes however, an average c_p and c_v may be used for most gases over the temperature range of interest.

Example IIa.2.5. We made a fit to data for c_p of air in the range of 360 R–2880 R and obtained:

$$(c_p)_{air} = 0.238534 - 6.20064 \times 10^{-6} T + 2.13043 \times 10^{-8} T^2 - 4.20247 \times 10^{-12} T^3$$

where temperature is in R and $(c_p)_{air}$ is in Btu/lbm·R. Find $(c_p)_{air}$ at $T = 80$ F according to the above fit.

Solution: At 80 F (540 R), specific heat of air at constant pressure according to the above equation becomes $(c_p)_{air} = 0.2421$ Btu/lbm·R. Compared with data in Table A.II.5(BU), the error is less than 1%.

Having c_v and c_p , we can calculate u and h by integrating $du = c_v dT$ and $dh = c_p dT$, respectively:

$$u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T) dT \quad \text{and} \quad h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T) dT$$

To simplify analysis, we may use an average value for c_v and c_p in the temperature range of interest:

$$c_v = \frac{\int_{T_1}^{T_2} c_v(T) dT}{T_2 - T_1} \approx \frac{c_v(T_1) + c_v(T_2)}{2} \quad \text{and} \quad c_p = \frac{\int_{T_1}^{T_2} c_p(T) dT}{T_2 - T_1} \approx \frac{c_p(T_1) + c_p(T_2)}{2}$$

where the arithmetic average applies to temperature ranges within which specific heat varies slightly. Having an average value for the specific heat we can calculate enthalpy, for example from:

$$h(T) - h(T_{ref.}) = c_p(T - T_{ref.})$$

Using $T_{ref.} = 0 \text{ R}$ (-460 F) and assuming $h(T_{ref.}) = 0$, then $h(T)$ can be written as $h(T) = c_p T$ where T is the absolute temperature in degrees Rankine.

3. Equation of State for Water

In this section, the equation of state for water is discussed following the definition of some pertinent terms.

3.1. Definition of Terms

Saturation temperature is the temperature at which boiling takes place at a given pressure.

Saturated liquid or vapor is a state of a substance at which change in phase takes place while the substance temperature remains constant. At saturation, the substance pressure is referred to as the *vapor pressure*. The vapor pressure is a function of temperature hence it remains constant during the phase change.

Subcooled or compressed liquid is a liquid phase of a substance, which exists at a temperature less than the saturation temperature corresponding to the substance pressure.

Superheated vapor is the vapor phase of a substance that exists at a temperature greater than the saturation temperature corresponding to the substance pressure.

Helmholtz function (a) is another thermodynamic property of a substance and is defined as $a = u - Ts$. The Helmholtz function has units of energy.

Gibbs function (g) is also a thermodynamic property of a substance and is defined as $g = h - Ts$. The Gibbs function has units of energy.

Maxwell relations are four well known thermodynamic equations written in terms of intensive properties. The Maxwell relations correlate temperature and entropy to other thermodynamic properties as follows:

$$Tds = du + Pdv \quad \text{IIa.3.1}$$

$$Tds = dh - vdP \quad \text{IIa.3.2}$$

$$sdT = -da - Pdv \quad \text{IIa.3.3}$$

$$sdT = -dg + vdP \quad \text{IIa.3.4}$$

where in these relations, a is the *Helmholtz function* (after Herman Ludwig von Helmholtz, 1821–1894) and g is the *Gibbs function* (after Josiah Willard Gibbs, 1839–1903). An example for the use of Maxwell's relations includes the calcula-

tion of entropy change of a system in terms of other thermodynamic properties. If we write Equation IIa.31 for an ideal gas as $Tds = c_v dT + R dT/T$ and then integrate it, we obtain the change in entropy for an ideal gas as:

$$s_2 - s_1 = R \ln \frac{v_2}{v_1} + \int_1^2 c_v \frac{dT}{T} \quad \text{IIa.3.5}$$

The specific heat of some gases, frequently used in common practice, are given in Table A.II.5. If the specific heat is taken as constant, the integral in Equation IIa.3.1 can be carried out to obtain:

$$s_2 - s_1 = R \ln \frac{v_2}{v_1} + c_v \ln \frac{T_2}{T_1} \quad \text{IIa.3.6}$$

We may apply Equation IIa.3.5 to an ideal gas and obtain a similar relation but in terms of pressure ratio.

Coefficient of volume expansivity (β) or thermal expansion coefficient is a measure of the change in specific volume with respect to temperature with pressure held constant. This coefficient is given as, $\beta = [(\partial v / \partial T)_P] / v = -[(\partial \rho / \partial T)_P] / \rho$. The coefficient of volume expansivity has the units of K^{-1} or R^{-1} .

Isothermal compressibility (κ) is a measure of change in specific volume with respect to pressure at constant temperature, $\kappa = -[(\partial v / \partial P)_T] / v$. It has the units of bar^{-1} or psi^{-1} . The minus sign is intended to maintain a positive value for κ regardless of the phase or the substance.

Isentropic compressibility (α) is a measure of change in specific volume with respect to pressure at constant entropy, $\alpha = -[(\partial v / \partial P)_S] / v$. It has the units of bar^{-1} or psi^{-1} . The minus sign is intended to maintain a positive value for κ regardless of the phase or the substance. Entropy is defined in Section 1.4.

3.2. Equation of State

Due to its availability and reasonably good physical properties, water is extensively used as a working fluid in practice. As such, water properties have been carefully measured, formulated, and tabulated. The tabulation of the thermodynamic properties of water is known as the steam tables, as presented in Tables A.II.1(SI) through A.II.4(SI) and A.II.1(BU) through A.II.4(BU) for SI and British units, respectively. Traditionally, thermodynamic properties in the steam tables are arranged with pressure and temperature as independent variables. These tables could have been arranged using any other two intensive properties such as specific volume and specific internal energy, as independent variables.

The functional relationship for water between P , T , and v is shown in Figure IIa.3.1(a). The single-phase states such as solid, liquid, and steam are identified in this figure. Also shown are two-phase regions such as liquid-vapor and solid-vapor. The projections of various regions of Figure IIa.4.1(a) on the P - T and

on the T - v surfaces are shown in Figures IIA.3.1(b) and IIIA.3.1(c), respectively. We examine these figures in more detail. Figure IIA.3.1(b) shows three distinct lines: the sublimation line, the fusion line, and the vaporization line. Pure substances at equilibrium generally exist either as solid, liquid, or gas. However, depending on the pressure and temperature two or even all of these three phases may coexist. For example, water at 32 F and 4.58 mm Hg (0.006 atm) may exists as ice, water, or steam or any combination of these phases at equilibrium. This specific point is known as the *triple point*. To further elaborate on the vaporization line, we consider a cylinder fitted with a piston. Initially, the cylinder contains superheated steam. As an example, steam can be at an absolute pressure of 18 psia and temperature of 250 F, as shown in Figure IIA.3.2 as State A.

Fehler! Keine gültige Verknüpfung.
(a)

Fehler! Keine gültige Verknüpfung.
(b)

Fehler! Keine gültige Verknüpfung.
(c)

Figure IIA.3.1. Pressure-temperature-volume plots for water (not to scale)

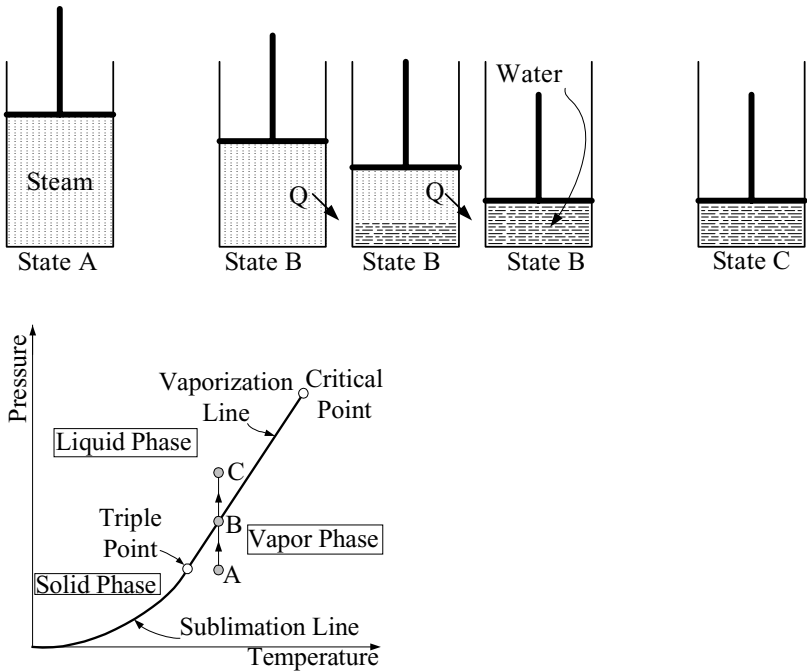


Figure IIA3.2. Steam condensation in an isothermal process

We now apply force on the piston, which increases steam pressure and temperature. To maintain temperature, the applying force on the piston must take place in an isothermal process. This is possible by allowing heat transfer from the cylinder to the surroundings, causing steam condensation. Upon condensation of all the steam in the cylinder, the pressure and temperature of state B reach 29.825 psia and 250 F, respectively. We may continue applying pressure on the piston and allowing heat transfer from the cylinder until state C is reached. For the numerical example, state C reflects a compressed or subcooled liquid at 250 F and a pressure greater than 29.825, say 34 psia.

Let's now examine an isobaric process with water in the cylinder being at state C at which $P = 34$ psia and $T = 250$ F (Figure IIa.3.3). We maintain the applied force on the piston but add heat to the water in the cylinder until water begins to boil. To maintain pressure, we let the piston move upward to accommodate the evaporation process and the expanding volume. We continue heating water until the last drop of water evaporates. This is state D where for our example, pressure is 34 psia and steam temperature has reached 257.58 F. Upon further heating with volume expansion, we reach state E at which steam is superheated. For the numerical example, state E is at 34 psia and a temperature higher than 257.58 F, say 265 F.

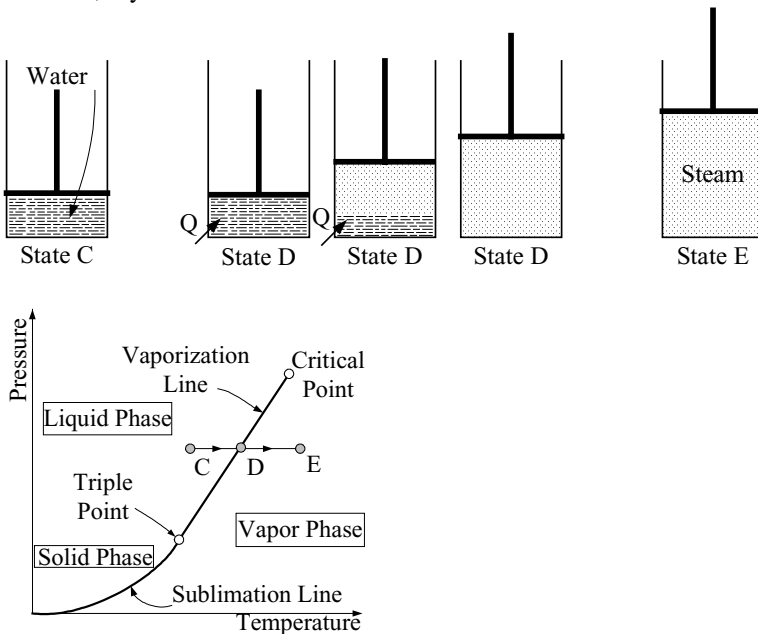


Figure IIa.3.3. Water evaporation in an isobaric process

Returning to Figure IIa.3.1, let us now examine Figure IIa.3.1(c). As shown in T - V diagram of Figure IIa.3.1(c), water goes through three major phases at a given pressure and rising temperature. On the left of the saturated-liquid line, water is subcooled otherwise known as compressed liquid. On the right of the saturation-vapor line, water is in the form of superheated vapor. A two-phase mixture exists between the two saturation lines with the mass of steam increasing from left to right. This is better determined by defining a steam static quality for a two-phase mixture:

$$x = \frac{\text{mass of steam}}{\text{mass of mixture}} = \frac{\text{mass of steam}}{\text{mass of steam} + \text{mass of liquid}}$$

When we refer to quality we generally mean static quality as defined above. Other definitions for quality are discussed in Chapter Va. Quality is zero on the saturation-liquid line and is unity on the saturation-vapor line. At any given point where A has a given steam quality of x , various thermodynamic properties are obtained as follows. We first read various saturated-liquid and saturated-vapor properties from the steam tables:

$$P_1 \rightarrow v_f \quad v_{fg} \quad v_g \quad u_f \quad u_{fg} \quad u_g \quad h_f \quad h_{fg} \quad h_g \quad s_f \quad s_{fg} \quad s_g$$

where properties of saturated liquid and saturated vapor are shown with subscripts f and g , respectively. Any property with subscript $_{fg}$ refers to the difference in values from saturated liquid to saturated vapor (i.e., $\pi_{fg} = \pi_g - \pi_f$ where $\pi = v, u, h, s$, etc.). In particular, h_{fg} represents the *latent heat of vaporization*. The latent heat by definition, is the energy stored in (or released from) a substance during a phase change, which occurs at constant pressure and temperature. For example, h_{fg} is that amount of heat required to vaporize saturated water to become saturated steam. Similarly, h_{fg} is that amount of heat, which is released by saturated steam to condense to saturated water. Having the saturated liquid and saturated vapor properties, we can calculate properties of a mixture of water and steam for given steam quality x as follows:

$$\begin{aligned} v &= v_f + x(v_g - v_f) = v_f + xv_{fg} \\ u &= u_f + x(u_g - u_f) = u_f + xu_{fg} \\ h &= h_f + x(h_g - h_f) = h_f + xh_{fg} \\ s &= s_f + x(s_g - s_f) = s_f + xs_{fg} \end{aligned}$$

Example IIa.3.1. Find properties of state A in Figure IIa.3.1(c) using $P_1 = 800$ psia (5.5 MPa) and $x_A = 0.7$.

Solution: From the steam tables A.II.1(BU) we find:

v_f (ft ³ /lbm)	v_{fg} (ft ³ /lbm)	u_f (Btu/lbm)	u_{fg} (Btu/lbm)
0.02087	0.54809	506.70	608.50
h_f	h_{fg}	s_f	s_{fg}

(Btu/lbm)	(Btu/lbm)	(Btu/lbm·R)	(Btu/lbm·R)
509.80	689.60	0.7111	0.7051
$v_A = 0.02087 + 0.7 \times 0.54809 = 0.4045 \text{ ft}^3/\text{lbm}$ $u_A = 506.7 + 0.7 \times 608.5 = 932.65 \text{ Btu/lbm}$ $h_A = 509.8 + 0.7 \times 689.6 = 992.52 \text{ Btu/lbm}$ $s_A = 0.7111 + 0.7 \times 0.7051 = 1.20 \text{ Btu/lbm·R}$			

Example IIa.3.2. Temperature and quality of a saturated mixture are given as 230 C and 85%, respectively. Find the thermodynamic properties for this mixture.

Solution: From the steam tables A.II.2(SI) at $T_{sat} = 230 \text{ C}$ we find:

v_f (m ³ /kg)	v_g (m ³ /kg)	u_f (kJ/kg)	u_g (kJ/kg)
1.2088E-3	0.07158	986.74	2603.90
h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kg·K)	s_g (kJ/kg·K)
990.12	2804.00	2.6099	6.2146

$$v = v_f + x(v_g - v_f) = 1.2088\text{E-}3 + 0.85 \times (0.07158 - 1.2088\text{E-}3) = 0.061 \text{ m}^3/\text{kg}$$

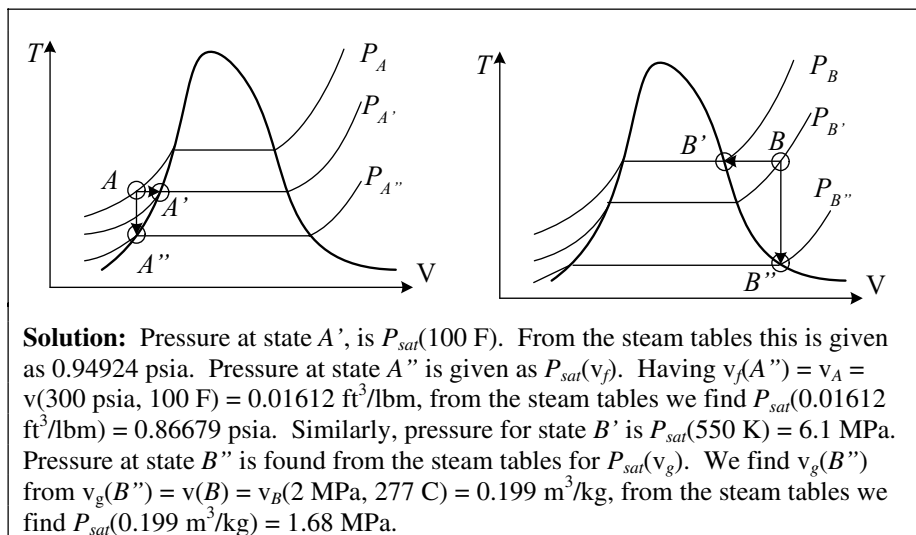
$$u = u_f + x(u_g - u_f) = 986.74 + 0.85 \times (2603.90 - 986.74) = 2,361.33 \text{ kJ/kg}$$

$$h = h_f + x(h_g - h_f) = 990.12 + 0.85 \times (2804.00 - 990.12) = 2,531.92 \text{ kJ/kg}$$

$$s = s_f + x(s_g - s_f) = 2.6099 + 0.85 \times (6.2146 - 2.6099) = 5.6739 \text{ kJ/kg·K}$$

Note that for the subcooled and superheated regions, any two intensive properties are sufficient to clearly define the state of water. These can be pressure and temperature, pressure and specific enthalpy, etc. While the same is true for the saturation region (i.e. the state of water is determined by having two independent properties), we cannot determine the state of water by having only pressure and temperature because, in the saturation region, these are functionally related and hence are not independent variables. In the saturation region, we are generally given pressure and quality, temperature and quality, pressure and enthalpy, etc.

Example IIa.3.3. State A in the left figure is subcooled water at $P = 300 \text{ psia}$ and $T = 100 \text{ F}$. Find the new pressure if the process is isothermal with final state being saturated water (A'). Also find the new pressure in a constant volume process with the final state being saturated water (A''). Repeat similar problem this time for state B in the figure on the right. State B is superheated steam at $P = 2 \text{ MPa}$ and $T = 277 \text{ C}$ with states B' and B'' being saturated steam.



Shown in Figure IIA.3.4 are the trends of saturated water and saturated steam enthalpies (h_f and h_g), water latent heat of vaporization (h_{fg}), and saturation temperature (T_{sat}) all as functions of pressure. Obtaining an equation for these or other saturation properties is much simpler than in the subcooled and superheated regions. This is due to the fact that in the saturated region we need to fit the curve to a function of a single variable as opposed to other regions which requires a curve fitting to functions of two variables. Even for the single variable function of the saturated region, a property often needs to be represented by more than one function in a piecewise fit to enhance the accuracy of the curve fit to data. Examples of curves fit to data are shown in Table A.II.6 of Appendix II. This appendix includes polynomial functions to represent P , v_f , v_{fg} , u_f , and u_{fg} in terms of temperature and are obtained by fitting curves to the steam tables data.

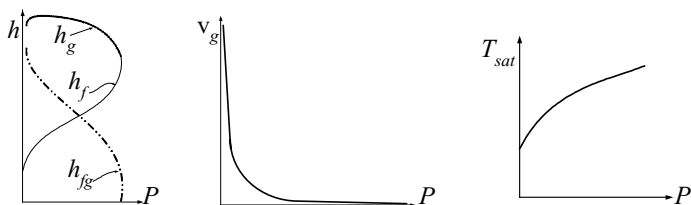


Figure IIA.3.4. Water enthalpy, steam specific volume, and saturation temperature versus pressure

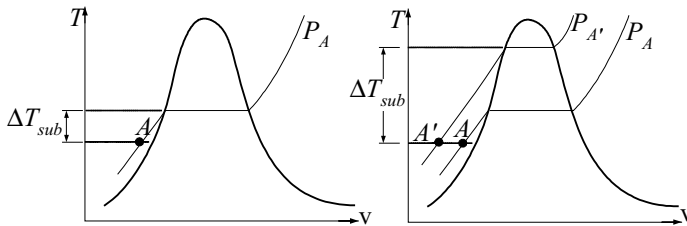
Example IIA.3.4. Consider a process where heat, mass, and work are exchanged with the surroundings. In the final state, u and v are known. Determine pressure and temperature of the final state.

Solution: The conservation equations of mass and energy are discussed later in

this chapter. For now, our purpose of presenting this example is to indicate that in thermohydraulic computer codes, mass (m) is obtained from the conservation equation of mass, total internal energy (U) from the conservation equation of energy, and volume (V) from the volume constraint. Specific internal energy (u) and specific volume (v) are obtained from $u = U/m$ and $v = V/m$, respectively. While the steam tables are traditionally arranged in terms of pressure (P) and temperature (T), it is generally the u and v that are calculated in the analysis. We should then find P and T from u and v by iteration with the steam tables. This is called the *pressure search* method. An example of such iteration is given by Program A.II.1 on the accompanying CD-ROM.

Below, we introduce the *degree of subcooling*, which is an indication of how far a system is from boiling. The degree of subcooling is an important parameter in pressurized subcooled systems, such as the primary side of a PWR, which must remain subcooled during normal operation.

Example IIa.3.5. Find the pressure required to maintain 50 C subcooling in a tank containing water at 200 C and 2 MPa.



Solution: The *degree of subcooling* is the difference between the saturation temperature and the actual temperature of a compressed liquid. At $P_A = 2$ MPa and 200 C, water is subcooled but the degree of subcooling is about $212 - 200 = 12$ C. To have a degree of subcooling increased to 50 C, we should find P_{sat} corresponding to $T = 200 + 50 = 250$ C which is about $P_{A'} = 3.97$ MPa.

Clapeyron equation is applicable to a phase change, which occurs at constant temperature and pressure. The latter is true since saturation pressure is a function of temperature. The Clapeyron equation (after Emil Clapeyron, 1799 - 1864) for a liquid-vapor phase change is:

$$\left(\frac{dP}{dT}\right)_{sat} = \frac{h_{fg}}{T v_{fg}} \quad \text{IIa.3.7}$$

From the Clapeyron equation, we can determine the latent heat of vaporization (h_{fg}), which cannot be directly measured from the PvT data. Note that T in Equation IIa.3.7 is the absolute temperature.

Clausius-Clapeyron equation is obtained by introducing simplifying approximations into the Clapeyron equation. The introduction of such approximations

limits the application of the Clapeyron equation to relatively low pressures where v_f is negligible as compared to v_g , which in turn can be approximated as $v_g = RT/P$, as explained in Section 2.2. Substituting into Equation Ila.3.7, we get:

$$\left(\frac{dP}{P}\right)_{sat} = \frac{h_{fg}}{R} \frac{dT}{T} \quad \text{Ila.3.8}$$

Upon integration, we find vapor pressure as a function of temperature:
 $\ln P_{sat} = -(h_{fg} / P_{sat})(1/T) + c$.

Example Ila.3.6. Calculate the latent heat of vaporization for water at $T = 212$ F.

Fehler! Keine gültige Verknüpfung.

Solution: To find h_{fg} , we need to determine the slope at $T = 212$ F. From the steam tables we find the following data:

T (F)	P (psia)	v_{fg} (ft ³ /lbm)
211	14.407	—
212	14.696	26.782
213	14.990	—

The slope becomes $(14.99 - 14.407)/(213 - 211) = 0.2915$ psi/F. Substituting in Equation Ila.3.7, we find:

$$h_{fg} = (dP/dT)T v_{fg} = [0.2915 \text{ lbf}/(\text{in}^2 \cdot \text{F})] \times (144 \text{ ft}^2/\text{in}^2) \times (212 + 460) \text{ R} \times 26.782 \text{ ft}^3/\text{lbm} = 755,471 \text{ ft} \cdot \text{lbf}/\text{lbm}$$

$$\text{Alternatively, } h_{fg} = (755,471 \text{ ft} \cdot \text{lbf}/\text{lbm}) / (778.17 \text{ ft} \cdot \text{lbf}/\text{Btu}) = 970.8 \text{ Btu}/\text{lbm}.$$

3.3. Determination of State

In this section, we noted that to determine the state of a substance we must have two independent intensive thermodynamic properties. In this regard, there are generally two cases that we have to deal with.

Case 1, P or T specified. In this case, either P or T and one more property (v , u , h , or s) are given. We find the state from the steam tables since one of the two known properties is either P or T . Having the saturation properties corresponding to the given P or T , we then make an assessment to see if the state is subcooled, saturated, or superheated. For example, if P & u are given to find the state, we first find $u_f(P)$ and $u_g(P)$ from the property tables. We then make the following comparison to find the thermodynamic state:

<u>Subcooled liquid</u>	<u>Saturated mixture</u>	<u>Superheated vapor</u>
$u < u_f(P)$	$u_f(P) < u < u_g(P)$	$u > u_g$

If both P and T are given and $T \neq T_{sat}(P)$ or alternatively $P \neq P_{sat}(T)$ then the state is either subcooled or superheated. This discussion is summarized in Table IIa.3.1.

Table IIa.3.1. Type of properties given for case 1

Subcooled, Saturated, Superheated			Saturated	Subcooled	Superheated
$T \ \& \ v$	or	$P \ \& \ v$	$T \ \& \ x$	$T \ \& \ P$ $T < T_{sat}(P)$	$T \ \& \ P$ $T > T_{sat}(P)$
$T \ \& \ u$	or	$P \ \& \ u$			
$T \ \& \ h$	or	$P \ \& \ h$	$P \ \& \ x$		
$T \ \& \ s$	or	$P \ \& \ s$			

Case 2, $P \& T$ not specified. If the two specified properties are not P and T , the state can not be readily determined from the steam tables. This is what was referred to in Example IIa.3.4 as the pressure search. Often in analysis we solve for $u \& v$ or $h \& v$ and would then have to find $P \& T$. In this case, we generally have to resort to iteration, an example of which is shown on the accompanying CD-ROM, Program A.II.1.

3.4. Specific Heat of Water

In many engineering applications, we may approximate values for thermodynamic properties such as v , u , and h for subcooled liquids using saturated liquid data at a specified temperature. This implies that such values are primarily a function of temperature and vary slightly with pressure at fixed temperature. We may extend this approximation to specific heat. Also note that using $du = c_v dT$, we may express the specific internal energy of water in terms of specific heat at constant volume as $\Delta u = u(T) - u_{Ref.} = c_v (T - T_{Ref.})$. Choosing the reference temperature for water as $T_{Ref.} = 32 \text{ F}$ at which $u_{Ref.} = 0$, we find $u(T) \approx c_v (T - 32)$. If we use an average value of $c_v = 1 \text{ Btu/lbm}\cdot\text{F}$ in the range of 32 F to 450 F, the internal energy of water in Btu/lbm becomes $u(T) \approx (T - 32)$ where T is in Fahrenheit. The largest error of less than 2.5% for the above temperature range occurs at 450 F.

4. Heat, Work, and Thermodynamic Processes

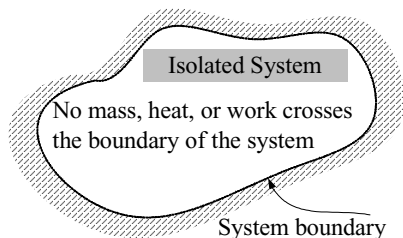
The laws of thermodynamics are known as the zeroth law, the first law, and the second law. The zeroth law is the basis for temperature measurement. This law states that if two systems are at the same temperature as a third system, the two systems would then have equal temperatures*. The first law deals with conservation of energy in a process. The second law governs the direction of the thermodynamic processes. The laws of thermodynamics are statements of fact and have no proof.

* The alternative expression for the zeroth law (also referred to as the third law of thermodynamics) is that at absolute zero, all perfect crystals have zero entropy.

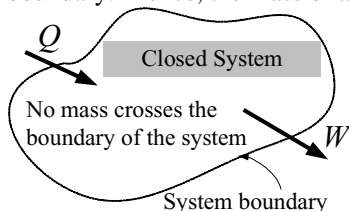
4.1. Definition of Terms

System, surrounding, and boundary are essential thermodynamics concepts, which allow us to study a substance, a region, or a process by considering it as a system and setting it apart from everything else known as the surroundings. Any interaction between the system and the surroundings takes place through the system boundary or the **control surface**. The boundary may be real or imaginary.

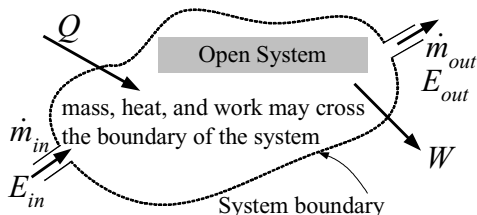
Isolated system is a system that does not have any interaction with its surroundings; hence neither mass nor energy can cross the boundaries of an isolated system.



Closed system, control mass allows transfer of energy but not mass through its boundary. Hence, the mass of a closed system is always constant.



Open system, control volume allows for transfer of mass and energy through the boundary. This is the most widely used means of analyzing thermodynamic processes. An open system may also be treated as a closed system by letting the system boundary change with the moving flow, and hence to encompass the same amount of mass at all times. Changes in the energy content of a closed system may also be due to such processes as thermal conduction, radiation, mechanical compression or expansion, and such fields as gravitational or electromagnetic.



Lumped parameter volume is a term applied to a system to emphasize the fact that there is only one temperature and pressure describing the entire system.

Distributed parameter volume, also known as subdivided volume, implies that a system is subdivided into several lumped volumes to increase the amount of detail we seek about the system while undergoing a process.

Adiabatic process refers to a thermodynamic process where there is no heat transfer to or from the system.

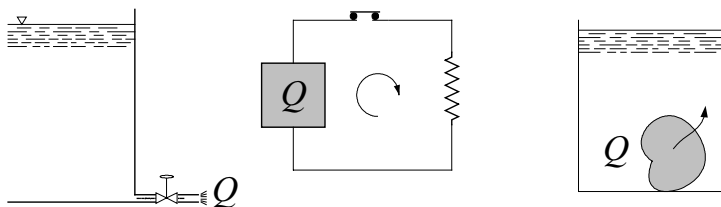
Reversible is an ideal process which, at the conclusion of the process, can be reversed to bring the system and its surroundings to the same exact condition as it was prior to the original process. Reversible processes are further discussed in Section 9 of this chapter.

Work, W is a form of energy transfer between a system and its surroundings if its net effect results in lifting a weight in a gravitational field. Various types of work are described in Section 4.3. The relation between heat (defined later in this section), work, and total energy is described in Section 6. This relation is generally referred to as the *energy equation* or *energy balance*. We assign a plus sign to the term representing work in the energy equation if work is delivered from the system to its surroundings. Otherwise we assign a minus sign. Work is not property of a system and must cross the boundary of the system. Work is expressed in J, kJ, or m·kgf in the SI system. In British units, work is given in Btu or less frequently used units of ft·lbf.

Power, \dot{W} is defined as the rate of energy transfer by work, $\dot{W} = dW/dt$. Since $W = F \times L$ where F is force and L is distance, then $\dot{W} = F \times V = (\Delta P \times A) \times V = \Delta P \times \dot{V} = (\Delta P/\rho) \times \dot{m}$. As described in Section 5, \dot{V} and \dot{m} are the volumetric flow rate and the mass flow rate, respectively.

Power is expressed in units of J/s, Watt (W, being the same as J/s), kilowatt (kW), megawatt (MW), gigawatt (GW), Btu/s, Btu/h, or horsepower (hp).

Heat, Q as a form of energy in transition is transferred due to a temperature gradient between two systems or a system and its surroundings, in the direction of decreasing temperature. The fact that heat flows solely due to temperature difference resembles the flow of water from a reservoir due to elevation difference or the flow of electricity from a capacitor (or from a battery) in an electric circuit due to potential difference. As shown in the left figure, when the valve is opened, water flows. The middle figure also shows that when the switch is turned on, electric current would be established in the circuit. Similarly, if we drop a hot block of copper into a bucket of colder water, heat flows from the copper to the water. By convention, if heat is delivered to a system we assign a plus sign to the term representing it in the energy equation. Conversely, if heat is transferred from the system to its surroundings, the sign is negative. Like work, heat is not a property of a system and it must cross the boundary of the system. Heat is expressed in J, kJ, or m·kgf in the SI system. In British units, heat is given in Btu or less frequently used units of ft·lbf.



4.2. Ideal Gas Processes

Processes involving ideal gases are referred to as polytropic when the following relation applies:

$$Pv^n = c \quad \text{IIa.4.1}$$

where c is a constant and n is the slope of the path plotted on the P - v coordinates, as shown in Figure IIa.4.1.

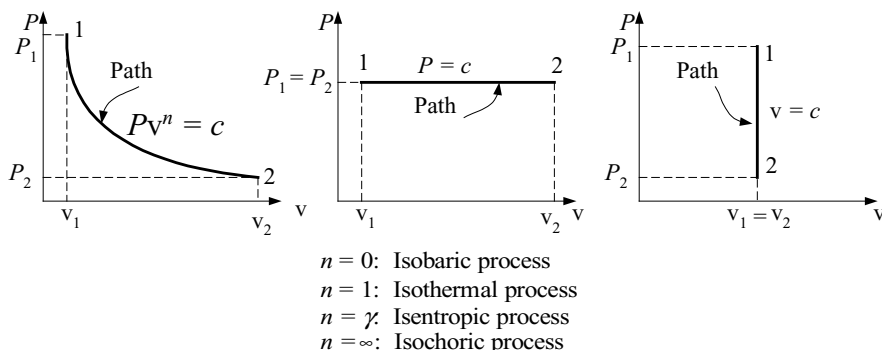


Figure IIa.4.1. Examples of Polytropic processes of an ideal gas

Special cases of the polytropic process include isobaric (constant pressure process), isochoric (constant volume process), isothermal (constant temperature process), and isentropic. Shown in Figure IIa.4.1 are also exponents of specific volume for special cases of isobaric, isochoric, isothermal, and isentropic processes. These exponents are derived by combining the equation of state ($Pv = RT$) and the polytropic process ($Pv^n = c$). To demonstrate, we take the derivative of the equation for the polytropic process and divide it by the equation for the polytropic process to obtain:

$$n = -\frac{v}{P} \frac{dP}{dv} \quad \text{IIa.4.2}$$

we now combine this equation with the equation of state for a specific case as demonstrated next.

Isothermal process. If we differentiate $Pv = RT$, we find $v dP + P dv = 0$ or $dv/dP = -v/P$. Substituting into Equation IIa.4.2, we find $n_{isotherm} = 1$.

Isobaric process. In this process, $P = c$ and $dP = 0$ hence, $n_{isobaric} = 0$.

Isochoric process is a constant volume process. For given mass, $v = c$ and $dv = 0$ hence, $n_{isochor} = \infty$

Isentropic process. An isentropic process is an adiabatic and reversible process hence constant entropy, $(dS)_{isentropic} = 0$. In an isentropic process for an ideal gas we have $n = \gamma$ where γ is given by $\gamma = c_p/c_v$. Hence, for an isentropic process we have $PV^\gamma = \text{constant}$. Combining with the equation of state, we find:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma \quad \text{IIa.4.3}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \quad \text{IIa.4.4}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{IIa.4.5}$$

4.3. Types of Work

The simplest type of work is the shaft-work needed to lift a weight in a gravitational field. Examples for various types of work are as follows: *compression work* delivered to a system consisting of a piston and a gas filled cylinder, *expansion work* delivered by a system consisted of a piston and a gas filled cylinder (Figure IIa.4.2), *electric work*, representing the movement of electric charge in a field of electric potential, *magnetic work*, representing the alignment of ions with the magnetic axes, *tension work* as in a stretched wire, *surface film work* against surface tension of a liquid, *rotating shaft work* such as that delivered by a turbine, and *shear work*, due to the existence of shear forces such as that required to pull a spoon out of a jar of honey. The relation for compression or expansion work in terms of pressure and volume can be obtained from the definition of work, which is the applied force times the displacement. For both cases of compression and expansion we have:

$$\delta W = F(dl) = (P \times A)(dl) = P(A \times dl) = PdV$$

Total work is obtained from: $W_{12} = \int_1^2 PdV$. If $P_1 = P_2 = P$ then $W_{12} = P(V_2 - V_1)$.

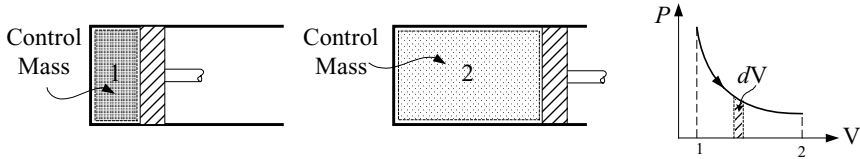


Figure Ila.4.2. Expansion work delivered by a system with moving boundary

Example Ila.4.1. The movement of the piston in the cylinder of Figure Ila.4.3 is frictionless. The spring is linear (displacement is proportional to the applied force) and is at its normal length when the piston is at the bottom of the cylinder in Figure Ila.4.3(a). We now introduce 5 kg of a mixture of water and steam to the cylinder. This causes cylinder pressure to reach 0.4 MPa at a quality of 0.2 in Figure Ila.4.3(b). At this stage, we add heat to the mixture. This causes expansion of the mixture and movement of the piston until it eventually reaches the stops in Figure Ila.4.3(c). Volume of the cylinder at this stage is $V_c = 1.0 \text{ m}^3$. We continue heating up the mixture until all water vaporizes and the mixture becomes saturated steam.

I) Show this process on a PV -diagram

II) Find pressure in stage c_2 where steam becomes saturated

III) Find pressure and steam quality at stage c_1 where piston just reaches the stops,

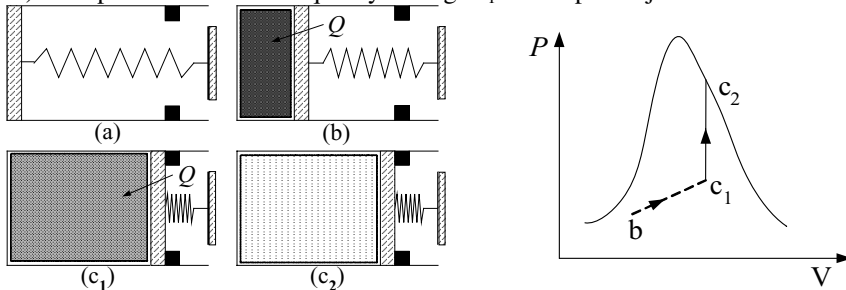


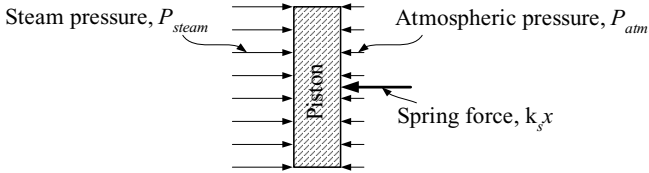
Figure Ila.4.3. Heat addition to a cylinder containing water mixture

Solution: I) This process is shown in the PV -diagram of Figure Ila.4.3. Initially, mixture has a quality of $x_b = 0.2$ at a pressure of 0.4 MPa (point b). Heat is then added to the mixture until at c_1 the piston reaches the stops. We continue heating the mixture until all water vaporizes and steam becomes saturated at c_2 . The process from state b to state c is a straight line because the spring is linear. The process from c_1 to c_2 is a vertical line, since pressure increases at a constant volume.

II) To find the pressure in state c_2 , we need to have two independent properties. We note that, in this state, steam is saturated, hence, quality is 100%. To find another independent property, we use the fact that we are dealing with a closed system hence, mass remains constant throughout states b and c. Having mass of $m = 5 \text{ kg}$, volume of $V_c = 1.0 \text{ m}^3$, and quality of $x_{c_2} = 1$, we find specific volume as $v_c = v_g = 1.0/5 = 0.2 \text{ m}^3/\text{kg}$. By interpolation in the steam tables, we find the corre-

sponding pressure of about $P_{c_2} = 1.00$ MPa.

III) To find pressure at state c_1 , we use the relation between displacement in the spring and the applied forces. This can be obtained from a free body diagram for the piston.



If we assume the spring constant is k_s and the cylinder cross sectional area is A , then from a force balance:

$$(P_{steam} - P_{atm})A = k_s x = k_s (V / A)$$

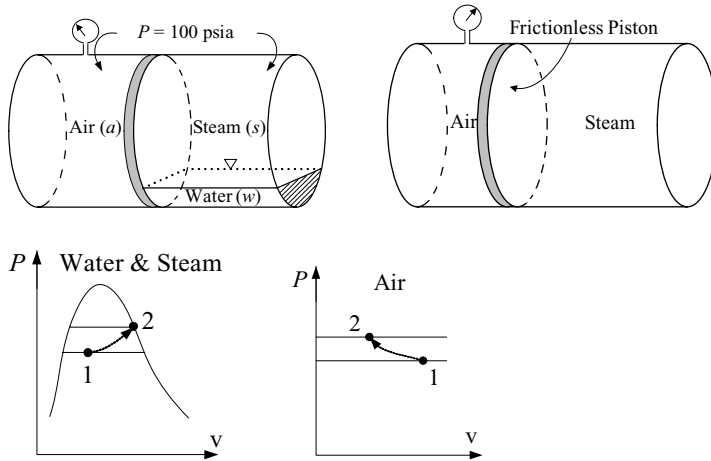
We are not given the spring constant k_s and the piston area A . We eliminate k_s and A by applying the above equation to both states b and c_1 and divide the results to obtain:

$$P_{c_1} - P_{atm} = (P_b - P_{atm})(V_{c_1} / V_b)$$

Having P_b , to find pressure at state c_1 , we need to first calculate volume at state b , which is obtained by multiplying the specific volume by the mixture mass. At $P_b = 0.4$ MPa, we find $v_b = 0.001084 + 0.2 (0.4625 - 0.001084) = 0.103$ m³/kg. Therefore, $V_b = mv_b = 5 \times 0.103 = 0.5156$ m³. Pressure at state c_1 is then found as $P_{c_1} = 0.1 + (0.4 - 0.1)(1/0.5156) = 0.682$ MPa. Note that we assumed $P_{atm} = 100$ kPa.

To find quality at c_1 , we use the fact that $v_{c_1} = v_{c_2} = 0.2$ m³/kg. Therefore, steam quality at c_1 becomes: $x_{c_1} = (v_{c_1} - v_f) / v_{fg}$. Using specific volumes at $P_{c_1} = 0.682$ MPa, $v_f = 0.001106$ m³/kg and $v_{fg} = 0.28$ m³/kg, we find $x_{c_1} = (0.2 - 0.001106) / 0.28 = 0.71$. Expectedly, the addition of heat has increased quality from 0.2 to 0.71. Further heat addition at constant volume increases quality to 100%.

Example IIa.4.2. A cylinder has a volume of 50 ft³. Half of the cylinder is filled with air and half with water and steam. The motion of the piston dividing the two chambers is frictionless while providing a perfect seal between the two chambers. The piston is a good heat conductor. Initially, the water volume is 5% of the total volume of the right chamber. We now add heat to both chambers until all of the water evaporates. Find the final pressure. Treat air as an ideal gas.



Solution: To find the final pressure we first need to determine all masses in the system. Since the motion of the piston is frictionless, both chambers are at the same pressure of 100 Psia. Since heat transfers from one chamber to the other through the piston, both chambers are also at the same temperature, $T = T_{sat}(P)$. We use subscripts a , s , and w for air, steam, and water, respectively. Subscript 1 is used for the initial equilibrium state (before heat is added to the system) and subscript 2 for final equilibrium state (after heat is added to the system). Throughout the entire process we can write the following five equations:

$$P_a = P_s = P, T_a = T_s = T_{sat}(P), \text{ and } V_a + (V_s + V_w) = 50$$

P_1 (psia)	T_1 (F)	v_{f1} (ft ³ /lbm)	v_{g1} (ft ³ /lbm)
100	327.82	0.01774	4.4310

The initial water mass is: $m_{w1} = V_{w1}/v_{f1} = (25 \times 0.05)/0.01774 = 70.462$ lbm.

The initial steam mass is: $m_{s1} = V_{s1}/v_{g1} = (25 \times 0.95)/4.4310 = 5.36$ lbm.

The initial air mass is:

$$m_{a1} = PV_{a1}/R_a T_1 = m_{a1} = (100 \times 144) \times 25 / [(1525/28.97) \times (327.82 + 460)] = 8.68 \text{ lbm.}$$

The final masses of water, steam, and air are $m_{w2} = 0$ lbm, $m_{s2} = m_{w1} + m_{s1} = 70.462 + 5.36 = 75.822$ lbm, and $m_{a2} = m_{a1} = 8.68$ lbm, respectively.

We find T_2 and P_2 by iteration:

We guess P_2 and find $T_2 = T_{sat}(P_2)$. Having P_2 and T_2 , we find $[v_{s2}(T_2)]_{Table}$. We also calculate v_{s2} from: $v_{s2}(T_2) = (50 - V_{a2})/m_{s2}$. Since $V_{a2} = (m_{a2} R_a T_2)/P_2$, therefore $[v_{s2}(T_2)]_{Calculated} = [50 - (m_{a2} R_a T_2)/P_2]/m_{s2}$.

The iteration is converged if $\left| [v_{s2}(T_2)]_{Table} - [v_{s2}(T_2)]_{Calculated} \right| \leq \epsilon$ where ϵ is the convergence criterion. Following this procedure, we find $P_2 = 755$ psia and $T_2 = 511.6$ F.

4.4. Work Involving an Ideal Gas

Using special processes for an ideal gas, we can find analytical expressions for work involving moving boundaries. If the process from the initial state (1) to the final state (2) as shown in Figure IIa.4.2 is such that the volume remains the same (**isochoric**), then $dV = 0$ and we find:

$$W_{1-2} = \int_1^2 P dV = 0$$

If the process in Figure IIa.4.2 is **isobaric**, then $P_1 = P_2$ and the work done from state 1 to state 2 is:

$$W_{1-2} = \int_1^2 P dV = P_1(V_2 - V_1)$$

If the process in Figure IIa.4.2 is **isothermal**, then from $PV = mRT$, we find $P_1V_1 = P_2V_2 = PV = \text{constant}$. Substituting in the integral for $P = P_1V_1/V$, we get:

$$W_{1-2} = \int_1^2 P dV = P_1V_1 \int_1^2 \frac{dV}{V} = P_1V_1 \ln \frac{V_2}{V_1}$$

In general, for a **polytropic** process from 1 to 2 in Figure IIa.4.2, we have $P_1V_1^n = P_2V_2^n = PV^n$. Substituting

$$W_{1-2} = \int_1^2 P dV = P_1V_1^n \int_1^2 \frac{dV}{V^n} = P_1V_1^n \left[\frac{V^{1-n}}{1-n} \right]_1^2 = \frac{P_2V_2 - P_1V_1}{1-n} \quad \text{IIa.4.4}$$

Example IIa.4.3. The cylinder in Figure IIa.4.2 is filled with air and initially is at $P_1 = 10$ kPa and $V_1 = 0.1$ m³. At the conclusion of a process the final air volume is $V_2 = 0.3$ m³. Find the work done by the piston if

a) the process is isobaric, b) the process is isothermal, c) the process is isentropic ($\gamma_{air} = 1.4$), and d) the process is polytropic with $n = 2$.

Solution:

a) In an isobaric process, $W_{1-2} = P_1(V_2 - V_1) = 10(0.3 - 0.1) = 2$ kJ.

b) For the isothermal process, $W_{1-2} = P_1V_1 \ln(V_2/V_1) = 10 \times 0.1 \times \ln(0.3/0.1) = 1.01$ kJ.

- c) For the isentropic process, $P_2 = P_1(V_1/V_2)^{1.4} = 10(0.1/0.3)^{1.4} = 2.148 \text{ kPa}$.
 $W_{1-2} = (P_2V_2 - P_1V_1)/(1 - n) = [2.148 \times 0.3 - 10 \times 0.1]/(1 - 1.4) = 0.89 \text{ kJ}$.
- d) For the polytropic process, $P_2 = P_1(V_1/V_2)^2 = 10(0.1/0.3)^2 = 1.111 \text{ kPa}$
 $W_{1-2} = (P_2V_2 - P_1V_1)/(1 - n) = [1.111 \times 0.3 - 10 \times 0.1]/(1 - 2) = 0.667 \text{ kJ}$.

5. Conservation Equation of Mass for a Control Volume

The conservation equation of mass, referred to as the *continuity equation*, is discussed in this section. Derivation of this equation is left to Chapter IIIa. The most intuitive way of comprehending this equation is to consider the liquid level in the tank of Figure IIa.5.1. Liquid may be added to the tank from various inlet ports and may be withdrawn from the tank through various outlet or exit ports. The liquid in this tank represents a control volume for mass or energy (note changes with time). To accumulate mass in the tank, the flow rate into the tank must exceed the flow rate out of the tank. Conversely, to deplete the tank, the flow rate out of the tank must exceed the flow rate into the tank. This intuitive statement about the rate of accumulation or depletion is also applicable to the conservation of energy, momentum, and angular momentum although its application to the conservation of mass and energy is easier to envision*. The mathematical representation of the above statement for conservation of mass is:

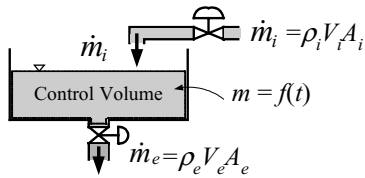


Figure IIa.5.1. Depiction of the rate equation

$$\sum_{inlet} \dot{m}_i = \sum_{exit} \dot{m}_e + \frac{dm_{C.V.}}{dt} \quad \text{IIa.5.1}$$

The subscript *C.V.* stands for control volume. The mass flow rate is related to the flow density, flow area, and flow velocity normal to the area as:

$$\dot{m} = \rho \vec{V} \cdot \vec{A} = \rho V A \quad \text{IIa.5.2}$$

It is customary to call $\dot{V} = VA$ *volumetric flow rate* and $G = \rho V$ *mass flux*. Thus, Equation IIa.5.2 can be expressed as:

* In Chapter VIe, we apply the same principle to derive the neutron transport equation.

$$\dot{m} = \rho VA = \rho \dot{V} = GA \quad \text{IIa.5.3}$$

The volumetric flow rate (\dot{V}) in SI units may be expressed as m^3/s , liter/s, etc. In British units, \dot{V} is usually given in terms of gallons per minute (gpm or GPM). Since $1 \text{ ft}^3 = 7.481 \text{ gallons}$, $1 \text{ ft}^3/\text{s} = 448.86 \text{ gpm}$.

Example IIa.5.1. Water at a rate of 54 GPM flows in a 3 inch-diameter pipe at 100 psia and 150 F. Find a) the volumetric flow rate, b) mass flow rate, c) mass flux, and d) flow velocity

Solution: At $P = 100 \text{ psia}$ and $T = 150 \text{ F}$: $v = 0.01634 \text{ ft}^3/\text{lbm}$, $\rho = 1/v = 1/0.01634 = 61.2 \text{ lbm/ft}^3$.

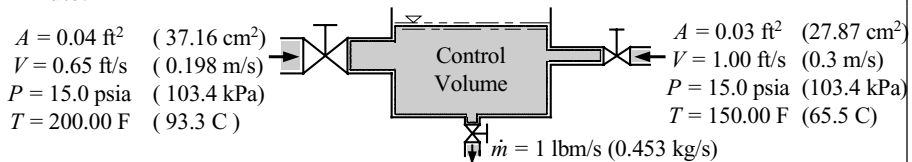
a) Volumetric flow rate of $\dot{V} = 54 \text{ GPM}$ is equivalent to $\dot{V} = 54/(7.481 \times 60) = 0.12 \text{ ft}^3/\text{s}$.

b) Mass flow rate becomes: $\dot{m} = \rho \dot{V} = 61.2 \times 0.12 = 7.36 \text{ lbm/s}$.

c) To find mass flux, we calculate the pipe flow area as $A = \pi d^2/4 = 3.14 \times (3/12)^2/4 = 0.049 \text{ ft}^2$. Having mass flow rate and flow area, mass flux becomes $G = \dot{m}/A = 7.36/0.049 = 150 \text{ lbm/ft}^2\cdot\text{s}$.

d) Finally, velocity is found from: $V = G/\rho = 150/61.2 = 2.45 \text{ ft/s}$. We could also find V from $V = \dot{V}/A$.

Example IIa.5.2. Water enters a mixing vessel from two inlet ports and leaves through one outlet port. Find the amount of water accumulated in the vessel in 1 minute.



Solution: We find the accumulated water by combining Equation IIa.5.1 with Equation IIa.5.2 and integrating:

$$m_2 - m_1 = [(\rho_1 V_1 A_1 + \rho_2 V_2 A_2) - \dot{m}_e] \Delta t$$

From the steam tables $v_1(15 \text{ psia} \& 200 \text{ F}) = 0.01664 \text{ ft}^3/\text{lbm}$ and $v_2(15 \text{ psia} \& 150 \text{ F}) = 0.01634 \text{ ft}^3/\text{lbm}$.

$\rho_1 = 1/v_1 = 1/0.01664 = 60.1 \text{ lbm/ft}^3$ (963 kg/m^3), $\rho_2 = 1/v_2 = 1/0.01634 = 61.2 \text{ lbm/ft}^3$ (980 kg/m^3)

$\Delta m = [(60.1 \times 0.65 \times 0.04) + (61.2 \times 1.00 \times 0.03) - 1] \times 60 = 143.92 \text{ lbm}$ (65.2 kg/s)

Steady flow steady state process: In the analysis of thermofluid systems, we often use the steady flow steady state process. Let's examine this process by con-

sidering the flow of gas through a gas turbine, representing our control volume. The steady flow condition requires that the mass flow rate of the gas entering the gas turbine to be equal to the mass flow rate of gas leaving the gas turbine. This also satisfies the steady state process during which $dm_{C.V.}/dt = 0$. However, as we will see in the next section, in a steady state process the rate of change of the energy of the control volume must also be zero, $dE_{C.V.}/dt = 0$.

Returning to the gas turbine example, it is true that the gas properties are changing as the gas flows through the blades of the turbine. That is to say the properties change spatially and there is indeed a profile for pressure, temperature, velocity, specific internal energy, and density from the entrance to the exit of the turbine. However, in the steady state condition, the spatial distribution of each property remains independent of time. On the other hand, in the unsteady state or transient situation, gas properties in the control volume not only have spatial variations but also vary with time. Hence in transient analysis, we must consider mass and energy accumulation or depletion in a control volume.

Let's now consider a case where in Figure IIA.5.1, the rate of either accumulation or depletion of mass is zero. For the conservation of mass, Equation IIA.5.1 predicts that:

$$\sum_{inlet} \dot{m}_i = \sum_{exit} \dot{m}_e \quad \text{IIA.5.4}$$

Equation IIA.5.4, also referred to as *mass balance*, is an intuitive implication of the steady flow condition.

6. The First Law of Thermodynamics

The first law of thermodynamics is a relation between heat, work, and the total energy of a closed system. The first law of thermodynamics, also known as the *conservation equation of energy* or simply the *energy equation* has the simplest mathematical form, if written for a closed system. We can intuitively obtain this equation by using logical deduction (i.e. we expect the total energy of an isolated system to remain constant). Total energy of a system, in general, is the summation of its internal, kinetic, and potential energies ($E = U + KE + PE$). Thus, for an isolated system, $dE = 0$ ¹. If the system is not isolated and heat and work are allowed to cross the moving boundary of the system (Figure IIA.4.2), we expect that addition of heat (Q) to this system and the production of some work (W) will result in a net change in the system total energy:

¹ The assertion that energy can neither be created nor destroyed is a fundamental law in classical mechanics. As discussed in Chapter VIe, on a sub-atomic basis, mass is a form of energy appearing in a nucleus as the binding energy. The binding energy, in turn, is manifested as the short term nuclear force. Thus, a more general statement is to say that mass-energy can neither be created nor destroyed.

$$\delta Q = \delta W + dE \quad \text{IIa.6.1}$$

In Equation IIa.6.1, heat is delivered from the surroundings to the system hence, a plus sign is used for the δQ term. In return, the system has delivered positive work. *Equation IIa.6.1 is the mathematical expression of the first law of thermodynamics.* It is important to remember that the sign convention described here applies only if the energy equation is written in the form of Equation IIa.6.1. In this equation, the term representing heat is in the left side and the terms representing work and the total energy are in the right side of the energy equation. Also note the distinction made in Equation IIa.6.1, between exact differential (shown by d) and non-exact differential terms (shown by δ). An exact differential, such as total energy, is independent of the process or path between the initial and the final equilibrium states. Rather, it depends only on the initial and the final state (also known as the end states) properties:

$$\int_1^2 dE = E_2 - E_1$$

On the other hand, heat and work in general cannot be integrated unless the process is known². For example, suppose we add heat to the control system in Figure IIa.4.2. By manipulating the movement of the piston, we can accomplish the path between two equilibrium states in various ways including an isobaric, an isothermal, or an isochoric process. Integrating Equation IIa.6.1, we obtain:

$$\int_1^2 \delta Q = \int_1^2 \delta W + (E_2 - E_1)$$

After integration, this equation is generally shown as:

$$Q_{12} = W_{12} + (E_2 - E_1)$$

where subscript 12 for Q and W emphasizes the change in the value of Q or W along the path from state 1 to state 2 while subscript 1 or 2 signifies condition at state 1 or at state 2. Note from this result that if system goes through a thermodynamic cycle, then $E_2 = E_1$ so that $\Delta Q = \Delta W$.

Example IIa.6.1. Find the following values for Example IIa.4.2: total heat delivered to the system, the compression work performed by the piston on the air, and the heat transfer to the air during the process.

² The only exception is when work is a result of the action of a conservative force such as the force applied by a linear spring. Mathematically, such forces are gradients of a scalar, hence:

$$W_{12} = \int_1^2 \vec{F} \cdot \vec{dr} = \int_1^2 \vec{\nabla} f \cdot \vec{dr} = \int_1^2 df = f_2 - f_1$$

Solution: To find total heat transfer, we apply the first law to the whole cylinder:

$$Q_{12} = W_{12} + U_2 - U_1$$

where $U_1 = U_{1a} + U_{1mix}$ and $U_2 = U_{2a} + U_{steam}$.

The initial mixture internal energy is: $U_{1mix} = m_{f1}u_{f1} + m_{g1}u_{g1} = 70.462(298.2) + 5.36(1105.2) = 26935.6$ Btu.

The final mixture internal energy is: $u_2 = u_g(P_2) = u_g(755 \text{ psia}) = 1116$ Btu/lbm.

$U_{2mix} = (70.462 + 5.36)(1116) = 84617$ Btu. Since for the whole cylinder $W_{12} = 0$:

$Q_{12} = (U_{steam} - U_{1mix}) + (U_{2a} - U_{1a}) = (84617 - 26935.6) + 8.68 \times 0.171(511.6 - 327.82) = 57954$ Btu.

To find the amount of work done by the piston we need to have the type of process. This is because work is a path-dependent function. However, the type of the process in which heat addition takes place is not specified. We, therefore, use an approximation as follows:

$$W_{12a} = \int_1^2 P dV = \sum_1^{14} P_i V_i$$

where we have divided the interval of $755 - 100 = 655$ psia to 13 equal intervals of 50 psi and one interval of 5 psi. We then use pressures of 750 psia, 700 psia, 650 psia, etc. and find corresponding volumes from the equation of state for air. Finding the area under the PV curve by numerical summation yields:

$W_{12a} = -966$ Btu. Applying the first law to the air compartment only we find:

$$Q_{12a} = W_{12a} + (U_{2a} - U_{1a}) = -966 + 8.68 \times 0.171(511.6 - 327.82) = -693 \text{ Btu.}$$

Returning to the first law of thermodynamics, if we apply Equation IIa.6.1 to a process which brings a system from its initial equilibrium state 1 to another equilibrium state 2, substitute for the total energy term, and integrate we obtain:

$$Q_{12} = W_{12} + (U_2 - U_1) + m(V_2^2 - V_1^2)/2 + mg(Z_2 - Z_1) \quad \text{IIa.6.2}$$

As discussed earlier, enthalpy is another extensive property of a system. If we substitute for compression work in terms of PV in Equation IIa.6.2, we see that the internal energy, U and PV appear together. If we represent this summation by $H = U + PV$, the working fluid enthalpy, we simplify thermodynamic computations involving the energy equation. Since enthalpy is an extensive property, the specific enthalpy, h , as an intensive property is obtained from $h = H/m = u + Pv$. As pointed out earlier, care must be exercised in calculating enthalpy from this relation using British units. For this reason, we may write $h = u + cPv$ where in British units $c = 144/778 = 0.185$ Btu/psia-ft³ for P in psia, v in ft³/lbm, and u and h in Btu/lbm, respectively.

6.1. Conservation Equation of Energy for a Control Volume

Turning now to the conservation equation of energy for a control volume, the most frequently used form is Equation IIa.3.12 derived in Chapter IIIa and repeated below:

$$\sum_i \dot{m}_i \left(h_i + V_i^2 / 2 + gZ_i \right) + \sum \dot{Q} + \dot{q}''' V = \sum \dot{W}_s + P\dot{V}$$

$$\sum_e \dot{m}_e \left(h_e + V_e^2 / 2 + gZ_e \right) + \frac{d}{dt} \left[m \left(u + V^2 / 2 + gZ \right) \right] \quad \text{IIa.6.3}$$

where we have considered only two work terms; the shaft work and the work associated with the change in the boundary of the control volume. Also, the rate of internal heat generation is explicitly accounted for. Equation IIa.6.3 as written for a control volume is equivalent to Equation IIa.6.1, written for a control mass.

Equation IIa.6.3 expresses the fact that the rate of change of total energy of a control volume depends on the rate of net energy entering and leaving the control volume as well as the rate of heat and work exchanged with the surroundings. The last term in the right side is the rate of change of total energy of the control volume, $dE_{C.V.}/dt$.

Equation IIa.6.3 in terms of the control volume enthalpy is obtained by substituting for $u = h - Pv$ to get:

$$\sum_i \dot{m}_i \left(h_i + V_i^2 / 2 + gZ_i \right) + \sum \dot{Q} + \dot{q}''' V =$$

$$\sum \dot{W}_s + V\dot{P} + \sum_e \dot{m}_e \left(h_e + V_e^2 / 2 + gZ_e \right) + \frac{d}{dt} \left[m \left(h + V^2 / 2 + gZ \right) \right] \quad \text{IIa.6.3-1}$$

We simplify Equation IIa.6.3 or IIa.6.3-1 for cases where changes in *K.E.* and *P.E.* energies are negligible:

$$\sum_i \dot{m}_i h_i + \sum \dot{Q} = \sum \dot{W}_s + P\dot{V} + \sum_e \dot{m}_e h_e + \frac{d(mu)}{dt} \quad \text{IIa.6.4}$$

$$\sum_i \dot{m}_i h_i + \sum \dot{Q} = \sum \dot{W}_s + V\dot{P} + \sum_e \dot{m}_e h_e + \frac{d(mh)}{dt} \quad \text{IIa.6.4-1}$$

where in these equations, $\sum \dot{Q}$ now includes three major terms; the rate of heat addition to the control volume from all external sources, the rate of internal heat generation in the control volume from all internal sources, and the rate of heat removal from the control volume:

$$\sum \dot{Q} = \sum \left(\begin{array}{c} \text{Rate of heat addition} \\ \text{from all external sources} \end{array} \right) + \sum \left(\begin{array}{c} \text{Rate of Internal} \\ \text{Heat Generation} \end{array} \right) - \sum \left(\begin{array}{c} \text{Rate of heat removal} \\ \text{from the control volume} \end{array} \right)$$

Steady state analysis: We now consider a case where in Figure IIa.5.1, the rates of either accumulation or depletion of mass and energy are zero. If there is no accumulation or depletion of mass and energy and the boundary is fixed or the pressure work is negligible, Equation IIa.6.3 predicts that:

$$\sum_i \dot{m}_i \left(h_i + V_1^2 / 2 + gZ_i \right) + \sum \dot{Q} = \sum \dot{W}_s + \sum_e \dot{m}_e \left(h_e + V_2^2 / 2 + gZ_e \right) \quad \text{IIa.6.5}$$

If the *K.E.* and *P.E.* of the entering and exiting streams are negligible, Equation IIa.6.5 simplifies to:

$$\sum_i \dot{m}_i h_i + \sum \dot{Q} = \sum \dot{W}_s + \sum_e \dot{m}_e h_e \quad \text{IIa.6.6}$$

where Equation IIa.6.6 is the steady state form of Equation IIa.6.4 with no pressure work. Equation IIa.6.6 can be further simplified if there is no heat or work transfer involved in a process. This is demonstrated in the next section by applying the conservation equations of mass and energy to several important thermofluid systems.

7. Applications of the First Law, Steady State

We now proceed to examine the application of the conservation equation of energy in various thermofluid systems such as nozzles, diffusers, turbines, compressors, pumps, heat exchangers, and valves. It must be emphasized that the application of the conservation equation of energy is generally associated with the use of the conservation equation of mass and the equation of state. We begin by introducing various terms.

7.1. Definition of Terms

Nozzles are flow paths with decreasing flow area, hence, increasing velocity in the flow direction (Equation IIa.5.2 for equal densities yields $V_2 = V_1 A_1 / A_2$) as shown in the left side of Figure IIa.7.1.

Diffusers are reverse nozzles, as shown in the right-hand side of Figure IIa.7.1. A diffuser is then a flow path with increasing flow area in the flow direction. Among various applications for nozzles and diffusers is flow measurement as with a flow orifice, a nozzle plate or by using the combined nozzle-diffuser in venturi meters.

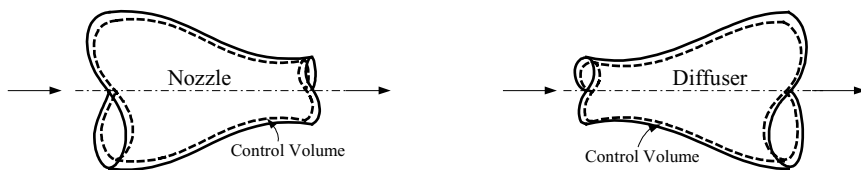


Figure IIa.7.1. Schematics of nozzle and diffuser

Turbines are mechanical devices that convert the energy of the working fluid to shaft work. Electric power is produced when the shaft work is delivered to the

rotor of a generator in a magnetic field. Turbines, if used in jet engines, deliver the shaft work to the compressor. The compressed air is then energized in the combustion chamber. A small percentage of the gas energy is used in the turbine to produce shaft work for the compressor. The rest leaves the jet engine in the form of rapid gas discharge to produce propulsion for the aircraft. A schematic of a turbine is shown in the left side of Figure IIa.7.2.

Compressors use shaft work to pressurize gases. Like turbines, the change in the potential energy from the inlet to the outlet of compressors is negligible. This is in comparison with the compression work delivered to the system. For well-insulated compressors, the rate of heat loss is also negligible. Schematic of a compressor is shown in the right side of Figure IIa.7.2.

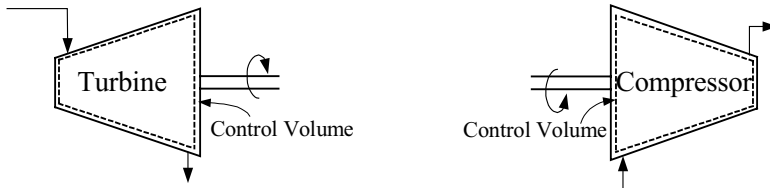


Figure IIa.7.2. Schematics of Turbine and Compressor

Pumps, like compressors, use shaft work to pressurize the working fluid, which is in the liquid phase. While the pumps and compressors perform identical functions, the difference between the density of gases and liquids results in drastic design differences for the device.

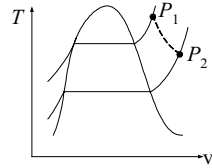
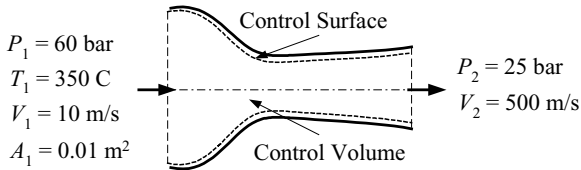
Heat exchangers are devices to transfer heat from a warmer to a colder fluid. Generally, in heat exchangers, the two streams of fluids do not mix; rather heat is transferred indirectly through tube walls or plates separating the streams. A heat exchanger in which the streams are mixed is called an open feedwater heater. Heat exchanger is a generic term which covers such diverse classes of devices as steam generators, condensers, radiators, boilers, intercoolers, and feedwater heaters.

Valves consist of a wide variety of devices to isolate or regulate flow or to control pressure. Gate valves isolate the flow, globe, ball, and butterfly valve regulate the flow, check valves prevent reverse flow, and safety and relief valves control pressure.

7.2. Conservation of Mass & Energy; Nozzles

The flow parameters that are most affected by passing through nozzles and diffusers are flow velocity and pressure. We, therefore, would have to consider change in kinetic energy. There is no work transfer and, if nozzles and diffusers are well insulated, there is also no heat transfer.

Example IIa.7.1. Steam enters a nozzle at 60 bar and 350 C and leaves through the diffuser at 25 bar at steady state condition. Use the data given below to find the outlet flow area, A_2 . The device is insulated.



Solution: We have 3 equations and 3 unknowns. The unknowns are mass flow rate, outlet temperature, and outlet flow area. The equations are the conservation equations of mass and energy as well as the equation of state. In the absence of heat and work, and in steady state steady flow conditions, Equation IIa.6.5 simplifies to:

$$h_1 + V_1^2 / 2 = h_2 + V_2^2 / 2$$

At 60 bar and 350 C for superheated steam we find $v_1 = 0.0423 \text{ m}^3/\text{kg}$ and $h_1 = 3043.67 \text{ kJ/kg}$. Substituting in the energy equation, we find, $h_2 = 3043.67 + (10^2 - 500^2)/2000 = 2918.7 \text{ kJ/kg}$. Having P_2 and h_2 , from the steam tables we obtain $v_2 = 0.0907 \text{ m}^3/\text{kg}$ and $T_2 = 264.5 \text{ C}$. From mass balance between inlet and outlet of the control volume we obtain:

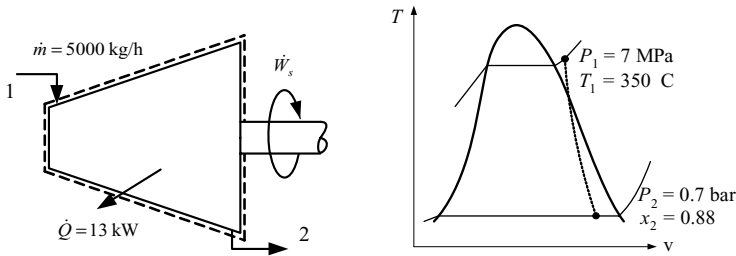
$$\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Substituting, $(10 \times 0.01) / 0.0423 = (500 \times A_2) / 0.0907$, we find $A_2 = 4.3 \text{ cm}^2$.

7.3. Conservation of Mass & Energy; Turbines

In turbines the potential and kinetic energy changes are generally negligible. Since turbines are insulated, the rate of heat transfer from turbines to the surroundings is also negligible compared to other terms in the energy equation.

Example IIa.7.2. Superheated steam enters a turbine at 7 MPa, 350 C, and a mass flow rate of 5000 kg/h. Steam leaves the turbine at 7 bar and a quality of 88%. The heat loss from the turbine is 13 kW. Calculate the rate of shaft work developed by the turbine.



Solution: To find the power output, we use Equation IIa.6.6:

$$\dot{m}_1 h_1 - \dot{Q} = \dot{W}_s + \dot{m}_2 h_2$$

We need to find the inlet and exit enthalpies. At state 1, for superheated steam we find $h_1 = 3016.6$ kJ/kg and at state 2, for a saturated mixture we find $h_{f2} = 376.47$ kJ/kg and $h_{fg2} = 2283.23$ kJ/kg. Having steam quality of $x_2 = 0.88$, $h_2 = 376.47 + 0.88 \times 2283.23 = 2359.31$ kJ/kg. From the conservation of mass we have $\dot{m}_1 = \dot{m}_2$. The rate of shaft work can then be calculated from Equation IIa.6.6 as:

$$(5000/3600) \times 3016.6 - 13 = \dot{W}_s + (5000/3600) \times 2359.3$$

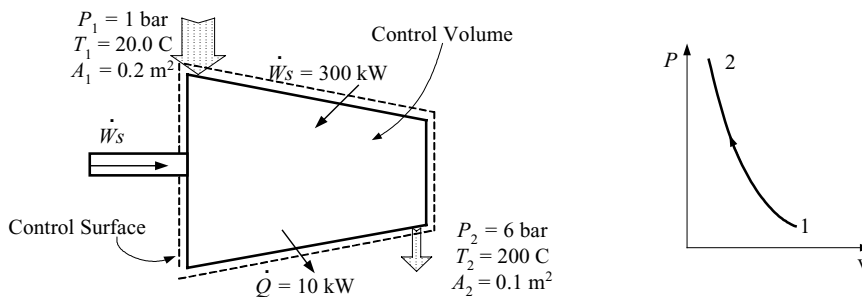
or $\dot{W} \approx 900$ kW. Note that the rate of heat loss to the surroundings is assigned a minus sign.

Ignoring heat loss, it is seen that the power produced by a turbine $\dot{W}_s = \dot{m}(h_i - h_e)$ depends on the mass flow rate and the change in enthalpy. To increase power for a fixed mass flow rate, we need to increase h_i and to lower h_e . Raising pressure, raising temperature, or raising both pressure and temperature can increase inlet enthalpy, h_i . The effects of raising P_i and T_i are discussed in Chapter IIb. The outlet enthalpy can also be reduced by lowering pressure at the outlet. This is the key feature in the design of condensers. Also note that in the design of steam turbines, it is important to ensure that dry steam flows in various stages of the turbine as moisture associated with the low-quality steam causes corrosion damage to turbine blades.

7.4. Conservation of Mass & Energy; Compressors

There are varieties of compressors to pressurize gases, including axial flow, reciprocating, rotary blower, sliding-vane, and screw-type rotary compressors.

Example IIa.7.3 . Find the mass flow rate delivered by a 300 kW air compressor with a compression ratio of 6. The maximum rate of heat loss from the compressor is estimated as 10 kW. Treat air as an ideal gas.



Solution: To find the mass flow rate, we consider a steady state condition and treat air as an ideal gas. Not having the velocities, we first assume that the change in kinetic energy is negligible. Equation Ila.6.6 simplifies to:

$$\dot{m}_1 h_1 - \dot{Q} = -\dot{W}_s + \dot{m}_2 h_2$$

From mass balance, we find that $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We also note that both \dot{Q} and \dot{W} terms have minus signs as heat is lost to the surroundings and work is delivered to the system. We then solve for mass flow rate:

$$\dot{m} = \frac{\dot{W}_s - \dot{Q}}{c_p(T_2 - T_1)} = \frac{300 - 10}{1.0035(200 - 20)} = 1.6 \text{ kg/s}$$

Now we can back calculate velocities. For this, we need to find specific volumes:

$$v_1 = \frac{R_u T_1}{MP_1} = \frac{8314(20 + 273)}{28.97 \times (1 \times 10^5)} = 0.84 \text{ m}^3/\text{kg} \text{ and}$$

$$v_2 = \frac{R_u T_2}{MP_2} = \frac{8314(200 + 273)}{28.97 \times (6 \times 10^5)} = 0.226 \text{ m}^3/\text{kg}$$

We find velocities from $V = \dot{m}v / A$. At the inlet, $V_1 = 1.6 \times 0.84 / 0.2 = 6.72 \text{ m/s}$. At the outlet $V_2 = 3.62 \text{ m/s}$.

The change in kinetic energy is $[(3.62)^2 - (6.72)^2]/2 = 16 \text{ kW/kg}$. This is about 9% of the change in enthalpy. We should then correct the mass flow rate in Equation Ila.6.6 by including the inlet and exit kinetic energies:

$$\dot{m}(h_i + \frac{V_i^2}{2}) - \dot{Q} = -\dot{W}_s + \dot{m}(h_e + \frac{V_e^2}{2})$$

Solving for the mass flow rate:

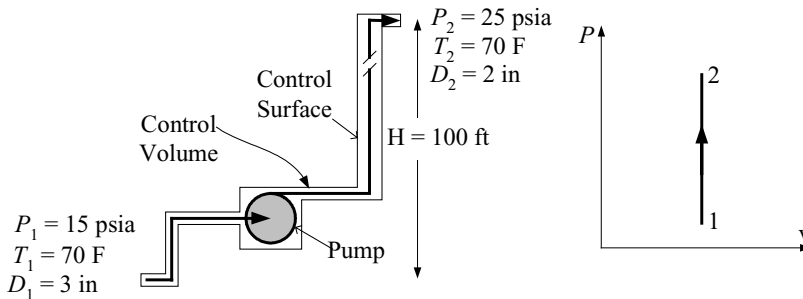
$$\dot{m} = \frac{\dot{W}_s - \dot{Q}}{c_p(T_e - T_i) + (V_e^2 - V_i^2)/2}$$

Substituting, we find the updated mass flow rate as $\dot{m} = 1.76 \text{ kg/s}$. We then update the change in kinetic energy and continue the iteration until we find the final mass flow rate as 1.81 kg/s .

7.5. Conservation of Mass & Energy; Pumps

The same fundamentals applied to compressors are applicable to pumps. However, pumps pressurize liquids with much higher density than gases. Therefore, the change in the potential energy of the liquid is substantial and must be considered in the energy equation.

Example IIa.7.4. Find the pumping power for steady flow of 360 gpm water in the pipeline below. Ignore frictional losses.



Solution: We should use Equation IIa.6.5:

$$\dot{m}(h_i + V_i^2/2 + gZ_i) - \dot{Q} = -\dot{W}_s + \dot{m}(h_e + V_e^2/2 + gZ_e)$$

For subcooled water, density and specific volume are practically functions of temperature. Hence, from the steam tables, $v_1 = v_2 = 0.01605 \text{ ft}^3/\text{lbm}$. Also, $h_1 = 38.09 \text{ Btu/lbm}$ and $h_2 = 38.12 \text{ Btu/lbm}$. Note that;

$$h_2 - h_1 \approx [h_f + v_f(P - P_{sat})]_2 - [h_f + v_f(P - P_{sat})]_1 = v_f(T)(P_2 - P_1)$$

We can verify this by substituting $h_2 - h_1 = 38.12 - 38.09 = 0.03 \text{ Btu/lbm}$.

Also $v_f(P_2 - P_1) = 0.01605(25 - 15) \times (144/778) = 0.0297 \text{ Btu/lbm}$. Hence, for pumps:

$$\Delta h_{\text{pump}} \cong v_f(T)\Delta P_{\text{pump}} \quad \text{IIa.6.7}$$

Let's now calculate the change in kinetic energy. For this, we need the inlet and outlet velocities. These can be found from $\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$. The volumetric flow rate is $360/(60 \times 7.481) = 0.8 \text{ ft}^3/\text{s}$. Inlet flow area is $A_1 = \pi D_1^2/4 = 0.049 \text{ ft}^2$. Therefore, the inlet velocity becomes $V_1 = 0.8/0.049 = 16.3 \text{ ft/s}$. The outlet flow area is $A_2 = \pi D_2^2/4 = 0.0218 \text{ ft}^2$ and $V_2 = 36.67 \text{ ft/s}$. The change in kinetic energy is:

$$(V_2^2 - V_1^2)/2 = [36.67^2 - 16.3^2]/(2 \times 32.2 \times 778) = 0.02 \text{ Btu/lbm}$$

We now find the change in potential energy;

$$g\Delta z = 32.2 \times 100/32.2 = 100 \text{ ft} \cdot \text{lbf} = 100/778 = 0.128 \text{ Btu/lbm}$$

Substituting into Equation IIa.6.5 and setting the rate of heat loss equal to zero, yields:

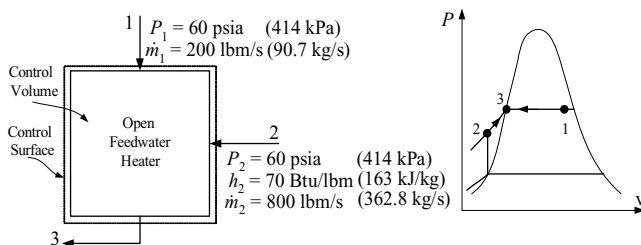
$$\dot{W}_s = (0.8/0.01605)[0.03 + 0.02 + 0.128] = 8.9 \text{ Btu/s}$$

A power of 8.9 Btu/s = 32036 Btu/h = 9.4 kW = 12.6 hp should be delivered to the pump. Actual power needed by the pump is more than 12.6 hp due to the mechanical and hydraulic losses in the pump, as discussed in Chapter VIc.

7.6. Conservation of Mass & Energy; Heat Exchangers

To demonstrate the conservation of mass and energy equations for heat exchangers, two examples are presented here. The first example deals with an open feedwater heater (also referred to as deaerator), in which the incoming streams mix.

Example IIa.7.5. Steam at quality x enters an open feedwater heater and after mixing with subcooled water, leaves as saturated water. Find the steam quality at the inlet port of the feedwater heater.



Solution: For steady state operation, we use Equation IIa.5.4 for mass balance:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Also from Equation IIa.6.6 for energy balance, with the rate of shaft work, the rate of heat loss, and the rate of change in the kinetic and potential energies set to zero we find:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

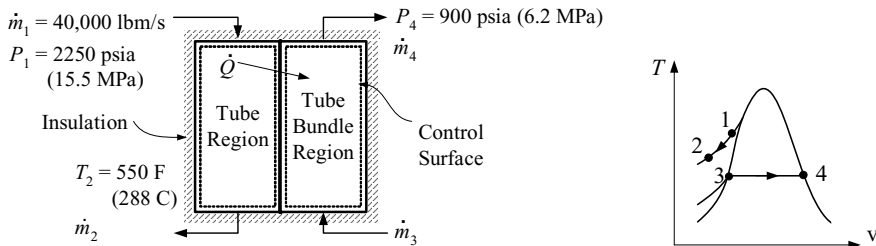
From the steam tables, $h_3 = h_f(60 \text{ psia}) = 262.2 \text{ Btu/lbm}$ and $h_{fg}(60 \text{ psia}) = 915.4 \text{ Btu/lbm}$. Hence:

$$200(262.2 + 915.4 x_1) + 800 \times 70 = (200 + 800) \times 262.2$$

Solving for the steam quality, we find $x_1 = 0.84$.

In the next example, we consider a steam generator of a PWR and will apply the conservation equations of mass and energy in conjunction with the equation of state to solve for the unknown parameters.

Example IIa.7.6. Subcooled water flows in the tubes of a steam generator in steady state condition. It leaves tubes with an enthalpy drop of 66.5 Btu/lbm. Find the rate of steam production and the rate of heat transfer from tubes for the given data.



Solution: We use Equation IIa.5.4 for mass balance and Equation IIa.6.6 for energy balance. There is no shaft work and the rate of change in the kinetic and potential energies is negligible. For the tube region:

$$\dot{m}_1 h_1 - \dot{Q} = \dot{m}_2 h_2$$

For the tube bundle region:

$$\dot{m}_3 h_3 + \dot{Q} = \dot{m}_4 h_4$$

Adding these equations and substituting from steady state continuity equation ($\dot{m}_1 = \dot{m}_2$ and $\dot{m}_3 = \dot{m}_4$), we obtain:

$$\dot{m}_1 (h_1 - h_2) = \dot{m}_3 (h_4 - h_3)$$

Since $h_1 - h_2 = 66.5$ Btu/lbm and $h_4 - h_3 = h_{fg}(900 \text{ psia}) = 669.7$ Btu/lbm.

Therefore, the rate of steam production is:

$$\dot{m}_3 = 40,000 \times 66.5 / 669.7 = 3972 \text{ lbm/s}, \approx 14.3 \times 10^6 \text{ lbm/h (1800 kg/s)}.$$

7.7. Conservation of Mass & Energy; Valves

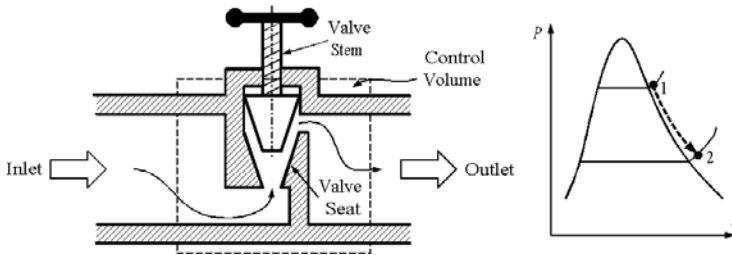
Valves that are used to control or throttle the flow rate accomplish this by introducing pressure drop to the flow. To analyze the effect of a valve on the flow, we may apply the first law of thermodynamics to a control volume taken around the valve. For this control volume, the change in potential energy is negligible. There

is also no work transfer and the rate of heat transfer is negligible, too. Combining the steady state mass and energy balance equations (Equation Ila.6.6), we obtain:

$$h_i + V_i^2 / 2 = h_e + V_e^2 / 2$$

In general, the kinetic energy terms are small compared with the enthalpies. Therefore, the process of flow going through valves and orifices can be considered isoenthalpic $h_i = h_e$

Example Ila.7.7. Steam at 900 kPa, 350 C, and a rate of 500 kg/s flows through a bypass pipe having a diameter of 1 m. The pipe is equipped with a partially open valve through which steam flows in a steady state condition. If the valve causes a 650 kPa pressure drop, find the steam temperature at the valve outlet.



Solution: First, we find h_1 (0.9 MPa & 350 C) = 3158 kJ/kg and $\rho_1 = 1/0.314 = 3.185 \text{ kg/m}^3$. We calculate velocities to show kinetic energies are small as compared with the fluid enthalpy. $A_1 = \pi 1^2 / 4 = 0.785 \text{ m}^2$ so that $V_1 = 500 / (0.785 \times 3.185) = 200 \text{ m/s}$. This amounts to $K.E. = V^2/2 = 200^2/2 = 20 \text{ kJ/kg}$. Thus; $h_2 \approx 3094 \text{ kJ/kg}$. Having $P_2 = 900 - 650 = 250 \text{ kPa}$ and $h_2 = 3094 \text{ kJ/kg}$ by iteration with the steam tables we find a steam temperature of about $T_2 \approx 308.5 \text{ C}$.

In the above example, we dealt with superheated steam entering and leaving the valve. If instead of steam, a liquid was flowing in the pipe and through the valve, we should use extra caution to ensure that the induced pressure drop to the flow would not result in flashing of the liquid. The flashing mechanism or partial vaporization of liquid would change the flow characteristics and may result in cavitation, as described in Chapter VIc.

7.8. Conservation of Mass & Energy; Heating Rigid Vessels of Constant Mass

Consider a rigid vessel containing a two-phase mixture. We want to study the heating of the mixture in this rigid vessel with no mass entering or leaving the system. As shown in Figure Ila.7.3, the vessel is initially at pressure P_1 . The control volume in this case can be viewed as a control mass. Since water and steam coex-

ist in the vessel at equilibrium, both water and steam are saturated at system pressure. Heat is now added until the vessel contains only saturated steam at pressure P_2 . The goal is to find the amount of heat added to the vessel.

The mixture mass and volume have remained the same throughout the heat up process. Therefore, $v_1 = v_2 = V/m$. This isochoric process is shown in the Pv diagram of Figure IIa.7.3.

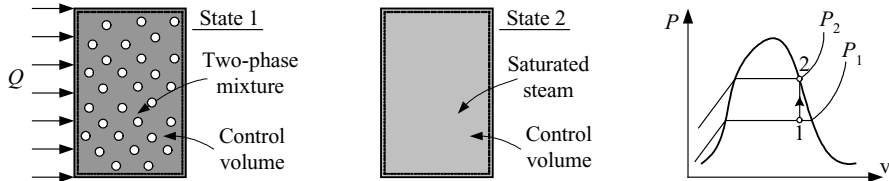


Figure IIa.7.3. Heating up a vessel containing saturated mixture

Before we embark on the solution, we must emphasize an important aspect of dealing with such problems. In Figure IIa.7.3, we have used one control volume to represent the entire mixture in the vessel. Stating that heat is transferred to the vessel implies that the separated regions of water and steam have no explicit meaning when represented with one control volume. Such *lumped* treatment of the problem does not allow specification of whether heat is added to the water or to the steam region. To obtain more details, we should at least assign one control volume to the water region and one to the steam region. To obtain even more information such as the temperature distribution in the water region, we must break down the water region into many more control volumes and apply the mass and energy equations to each control volume. Thus the allocation of only one control volume to the entire vessel implies that water and steam are homogeneously mixed at a given steam quality. This topic is discussed in more details in Section 5 of Chapter VI.

Returning to the heat up of the mixture in a rigid vessel, in order to find the amount of heat transfer to the vessel (Q), we must find at least two properties at state 2 in Figure IIa.7.3. In the case of Figure IIa.7.3 where only heat is added to the system, the vessel can be treated as a closed system. Thus, from the continuity equation we find that $m_2 = m_1 = m$. The first state property is v_2 since we know V and m hence, $v_2 = V/m$. The second property is obtained from the fact that fluid is saturated steam at P_2 (i.e., the steam quality at state 2 is $x_2 = 100\%$). If we have a function for $v_2 = f(P_2)$, we can solve for P_2 . Otherwise, we use the steam tables for $P_2 = P_g(v_g = v_2)$.

Having thermodynamic properties of both states 1 and 2, we can find heat transfer from the conservation equation of energy. The energy equation IIa.6.2 can be simplified for the following reasons. First, the work term drops as the closed system also has rigid boundary. Second, the kinetic energy terms drop as the system is at rest. Finally, the change in the potential energy (due to the change in mixture density) is negligible compared with the change in the internal energy. Therefore, Equation IIa.6.2 becomes:

$$Q_{12} = U_2 - U_1$$

It must be emphasized that, in practice, tanks must be equipped with pressure-relief valves as heating up an isolated tank would eventually lead to catastrophic failure of the tank. Rapid pressurization occurs if tanks are filled with liquids due to the lack of compressibility of the so-called “water-solid” systems.

Example Ila.7.8. A tank of 1500 ft^3 (42.5 m^3) contains steam at $P_1 = 1000 \text{ psia}$ (7 MPa) and $x_1 = 0.25$. Heat is added to the tank until steam quality becomes $x_2 = 100\%$. Find the heat transfer to the tank.

Solution: This is a closed system for which mass remains constant during the heat up process. Also the tank is rigid so volume remains constant. Hence, $v_1 = v_2$. To find mass and initial internal energy, we obtain:

P (psia)	v_f (ft ³ /lbm)	v_{fg} (ft ³ /lbm)	u_f (Btu/lbm)	u_{fg} (Btu/lbm)
1000	0.02159	0.42436	538.6	110.4

Hence, $v_1 = 0.02159 + 0.25(0.42436) = 0.12768 \text{ ft}^3/\text{lbm}$. This gives $m = V/v_1 = 1500/0.12768 = 11748 \text{ lbm}$. Also $u_1 = 538.6 + 0.25(1110.4) = 816.2 \text{ Btu/lbm}$. Since $v_2 = v_1 = 0.12768$ and the final state is saturated steam; $P_2 = P_g(v_2) = P_g(v = 0.12768 \text{ ft}^3/\text{lbm})$

A search in the steam tables for $v_g = v = 0.12768 \text{ ft}^3/\text{lbm}$ gives $P_2 = 2530 \text{ psia}$. At this pressure, $u_2 = u_g(P = 2530 \text{ psia}) = 1030 \text{ Btu/lbm}$. Hence, $Q = m(u_2 - u_1) = 11748(1030 - 816.2) = 2.5\text{E}6 \text{ Btu}$ ($2.64\text{E}6 \text{ kJ}$).

Could have we solved this problem if we were only told that state 2 was superheated steam at $P = 1000 \text{ psia}$?

7.9. Conservation of Mass & Energy; Heating Rigid Vessels at Constant Pressure

Let's now consider boiling water in a rigid vessel as shown in Figure Ila.7.4. The vessel is equipped with a control valve to discharge steam and maintain pressure at a desired value. To determine the steaming rate, we allocate two control volumes to the water and the steam regions. The makeup water, also known as *feed-water* is added to the vessel to maintain water level at a desired value. If there was no make up water to replenish the loss of inventory, the vessel would dry out. Heating up water in a vessel without providing any makeup water constitutes a transient problem. To have a heat addition process at steady flow and steady state condition, makeup water is added so that the mass flow rate of steam becomes exactly equal to the mass flow rate of the makeup water. This is shown in Figure Ila.7.4(a). The make up water may be subcooled as shown in Figure Ila.7.4(b) or saturated as shown in Figure Ila.7.4(c). If the makeup water is subcooled some of the heat is used to bring the subcooled water to saturation.

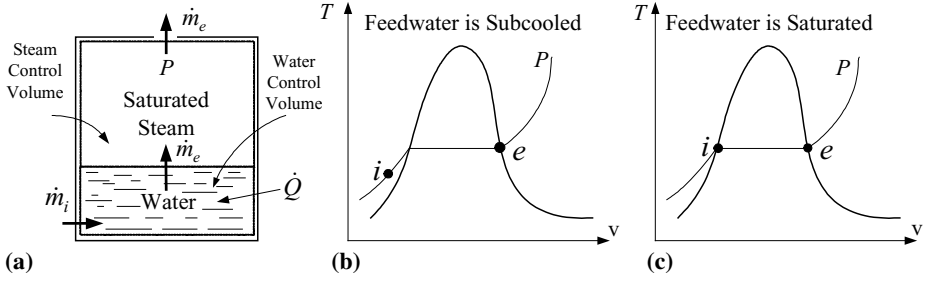


Figure IIa.7.4. Adding heat to a vessel at constant pressure

From the conservation equation of mass for steady flow and steady state conditions, $\dot{m}_i = \dot{m}_e = \dot{m}$. Also from the conservation equation of energy, $\dot{m}_i h_i + \dot{Q}_i = \dot{m}_e h_e$. Therefore, the steaming rate is found as:

$$\dot{m} = \frac{\dot{Q}}{h_g(P) - h_i} \quad \text{IIa.7.1}$$

As shown in Figure IIa.7.2-c, if the feedwater is saturated at pressure P ($h_i = h_f$), Equation IIa.7.1 reduces to:

$$\dot{m} = \frac{\dot{Q}}{h_{fg}(P)} \quad \text{IIa.7.2}$$

Note that, for a given rate of heat transfer, the steaming rate at a higher pressure is higher than the steaming rate at a lower pressure. This is due to the decrease in the latent heat of vaporization of water as pressure increases (Figure IIa.3.4).

8. Applications of the First Law, Transient

In practice, prior to establishment of steady state conditions, unsteady state or transient operation prevails. Transient operation can also be imposed on a system operating at a steady state condition. Consider for example, the steady flow of steam in a pipe when a fully open valve is throttled to a new partially open position. The flow of steam goes through a transient to reach a new steady state condition corresponding to the new position of the valve. We solve transient problems similar to problems for steady state condition by using the conservation equations of mass and energy as well as the equation of state. For a process that brings the control volume from state 1 at time t_1 to state 2 at time t_2 , we find the mass at state 2 by integrating Equation IIa.5.1:

$$\int_{t=t_1}^{t=t_2} \left(\sum_{inlet} \dot{m}_i \right) dt = \int_{t=t_1}^{t=t_2} \left(\sum_{outlet} \dot{m}_e \right) dt + [m(t_2) - m(t_1)] \quad \text{IIa.8.1}$$

Example IIa.8.1. Water is flowing in a 6 cm inside diameter pipe at a velocity of 0.75 m/s. The pipe discharges into an initially empty tank having a volume of 0.3 m³. How long will it take to fill the tank?

Solution: We first find the mass flow rate of water at atmospheric condition ($v = 0.001 \text{ m}^3/\text{kg}$) as:

$$\dot{m}_{in} = VA/v = 0.75 \times [\pi(0.06)^2/4]/0.001 = 2.12 \text{ kg/s.}$$

From Equation IIa.5.3 we have; $dm/dt = \dot{m}_{in} - \dot{m}_{out} = 2.12 - 0 = 2.12 \text{ kg/s.}$

Hence, $dm = 2.12dt$. Integrating: $m_2 - m_1 = 2.12t$. Since $m_1 = 0$, and $m_2 = V_{\text{tank}}/v = 0.3/0.001 = 300 \text{ kg}$, we find $t = 300/2.12 = 141 \text{ s.}$

Similarly, we find the internal energy of the control volume at state 2 by integrating Equation IIa.6.4:

$$\int_{t=t_1}^{t=t_2} \left(\sum_{inlet} \dot{m}_i h_i \right) dt - \int_{t=t_1}^{t=t_2} \left(\sum_{inlet} \dot{Q} \right) dt = \int_{t=t_1}^{t=t_2} \left(\sum \dot{W}_s \right) dt + \int_{t=t_1}^{t=t_2} \left(\sum_{outlet} \dot{m}_e h_e \right) dt + [m(t_2)u(t_2) - m(t_1)u(t_1)] \quad \text{IIa.8.2.}$$

ignoring changes in the *K.E.* and *P.E.* In the following examples, simple transient cases involving filling and draining containers are discussed. These include both cases of liquid and gas.

8.1. Dynamics of Mixing Tanks

Shown in Figure IIa.8.1 is a simple case of simultaneous filling and draining a heated mixing tank at atmospheric pressure. The tank is fed through several inlet ports. The shaft work is performed by the mixer and a heater may add heat to the water in the tank. Using the conservation equations of mass and energy, we can obtain two parameters in terms of other known data. For example, if the inlet mass flow rates, inlet enthalpies, mixer power, and the heater power are specified, we can solve for the mass and temperature of the tank water versus time. To perform this analysis, we make several assumptions: a) negligible *K.E.* and *P.E.* changes, b) perfect and instantaneous mixing, c) subcooled water in the tank throughout the process, d) no chemical reactions, e) no heat loss from the tank,

and f) constant tank pressure throughout the process. Applying these assumptions, Equation IIa.6.4-1 becomes:

$$\sum_1^N \dot{m}_i h_i + \dot{Q} = -\dot{W}_s + \dot{m}_e h_e + \frac{d(mh)_{C.V.}}{dt}$$

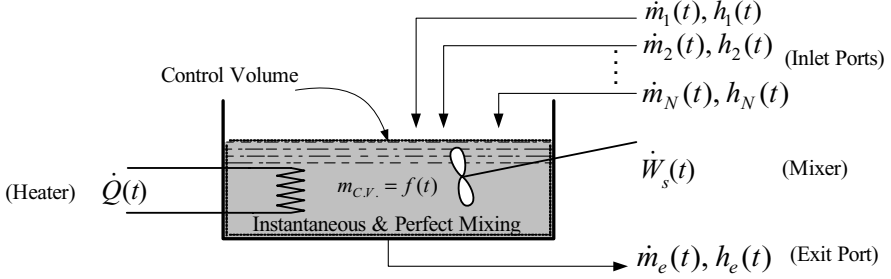


Figure IIa.8.1. Schematic of a simple mixing tank

We now take the derivative of the last term in the right side and substitute from Equation IIa.5.1 to find

$$\frac{dh_e}{dt} + \left(\frac{1}{m_{C.V.}} \sum_1^N \dot{m}_i \right) h_e = \frac{\dot{Q} + \dot{W}_s + \sum_1^N \dot{m}_i h_i}{m_{C.V.}} \quad \text{IIa.8.3}$$

where we used the perfect mixing assumption, which implies that $h_e = h_{C.V.}$. The mass of water in the tank is a function of time. For example for constant mass flow rates into and out of the tank, Equation IIa.5.1 predicts that the tank water mass varies linearly with time:

$$(m)_{C.V.} = (m_o)_{C.V.} + \left(\sum_i \dot{m}_i - \dot{m}_e \right) t$$

where $(m_o)_{C.V.}$ is the initial mass of water in the tank. Upon substitution of $(m)_{C.V.}$ into Equation IIa.8.3, we obtain a linear first-order differential equation for h_e . A general solution to such differential equations is given by Equation VIIb.2.4. Depending on the complexity of the functions representing the heater power, the shaft work, the inlet and exit mass flow rates, we may have to resort to numerical solutions, from which we obtain:

$$\text{Explicit: } h_e^{n+1} = h_e^n + \frac{\dot{Q}^n + \dot{W}_s^n + \sum_1^N \dot{m}_i^n (h_i^n - h_e^n)}{(m^n / \Delta t)};$$

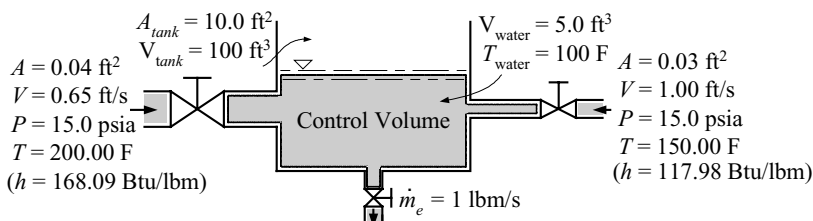
$$\text{Implicit: } h_e^{n+1} = h_e^n + \frac{\dot{Q}^n + \dot{W}_s^n + \sum_1^N \dot{m}_i^n (h_i^n - h_e^n)}{(m^n / \Delta t) + \sum_1^N \dot{m}_i^n}$$

where n is a time step index. In the above solution, a *semi-implicit* scheme is used (see Chapter VIIe and Problem 101).

Recall that for subcooled water $dh \cong c_p dT$. Assuming constant specific heat and substituting in the explicit scheme, for example, we find the water temperature at every time stop as:

$$T_e^{n+1} = T_e^n + \frac{\dot{Q}^n + \dot{W}_s^n + \sum_1^N \dot{m}_i^n (T_i^n - T_e^n)}{(m^n / \Delta t) c_p} \quad \text{IIa.8.4}$$

Example IIa.8.2. Flow enters a fully insulated tank from two inlet ports and leaves through one outlet port. Find water level in 30 minutes. Assume instantaneous and perfect mixing. Tank volume is 100 ft^3 , cross sectional area is 10 ft^2 , initial water volume is 5 ft^3 , and initial water temperature in the tank is $T_{\text{tank}} = 100 \text{ F}$.

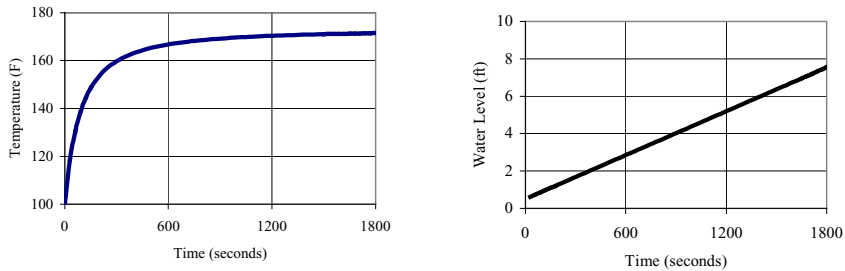


Solution: To find water level as a function of time, we first need to find water volume by dividing the mass of water by the density of water in the tank. Water level is then obtained by dividing water volume by the tank cross sectional area. Since there is no heat or work transfer to or from the control volume, temperature of water at each time step is found from the simplified form of Equation IIa.8.4:

$$T_e^{n+1} = T_e^n + \frac{\rho_1 V_1 A_1 (T_1 - T_e^n) + \rho_2 V_2 A_2 (T_2 - T_e^n)}{(m^n / \Delta t) c_p}$$

where we assumed constant specific heat. Initially (i.e., at time zero $n = 0$), the temperature, mass, and level of the tank water are $T_e^{n=0} = 100 \text{ F}$, $m^{n=0} = 5 \text{ ft}^3 \times 62 \text{ lbm/ft}^3 = 310.56 \text{ lbm}$, and $L^{n=0} = 0.5 \text{ ft}$, respectively. We now choose a time step size of $\Delta t = 0.1 \text{ s}$, for example, and find $T_e^{n=1}$ at $t = 0 + 0.1 = 0.1 \text{ s}$. Having found $T_e^{n=1}$, we proceed to find $T_e^{n=2}$ for time $t = 0.2 \text{ s}$. We continue this process until $t = 1800 \text{ s}$. The FORTRAN program representing the numerical solution is included on the accompanying CD-ROM. From the program we find that in 30

minutes, water temperature reaches to about 171.4 F at a level of 7.5 ft from the bottom of the tank, as plotted in the figure.



8.2. Charging Rigid Vessels (Fixed C.V.) with Gas

Shown in Figure IIa.8.2(a) is a tank connected to a charging line carrying pressurized gas at a known temperature. Initially, the intake valve is closed and the tank containing the same gas is at pressure P_1 and temperature T_1 . Figure IIa.8.2(b) shows the condition in the tank after the intake valve is opened. Figure IIa.8.2(c) shows the final state when the intake valve is closed. For a given initial condition of the vessel and the inlet enthalpy of the filling gas, we identify two cases to solve. In case A, having final pressure in the tank, we want to find the final gas temperature and the average gas mass flow rate entering the tank. In case B, for a given average mass flow rate of the filling gas, we want to find final pressure and temperature of the gas in the tank.

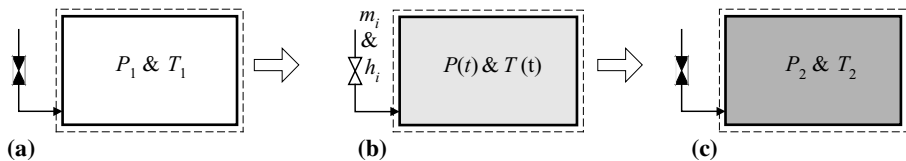


Figure IIa.8.2. Charging rigid vessels with gas and the associated control volume

Case A. For Given P_2 , Find T_2 and $\bar{\dot{m}}_i$:

As shown in Figure IIa.8.2, the control volume representing the rigid tank contains air and is initially at pressure P_1 and temperature T_1 . The intake valve is opened to allow the flow of air into the tank from a high-pressure source. The valve is closed when the pressure in the tank reaches the specified value of P_2 . The goal is to find the average mass flow rate of air entering the tank during the charging process.

In this case, there is no heat transfer, no shaft work, and no mass leaving the control volume. Therefore, Equation IIa.6.6 simplifies to:

$$\frac{d(mu)_{C.V.}}{dt} = \dot{m}_i h_i$$

From the conservation equation of mass (Equation IIa.5.1) we have $dm_{C.V.}/dt = \dot{m}_i$. Substituting, we get:

$$\frac{d(mu)_{C.V.}}{dt} = h_i \frac{dm_{C.V.}}{dt}$$

Treating air as an ideal gas ($du = c_v dT$), h_i as a constant, and integrating from the initial state (P_1 and T_1) to the final state where P_2 is specified yields:

$$m_2 c_v T_2 - m_1 c_v T_1 = h_i \int_1^2 dm_{C.V.} = h_i (m_2 - m_1)$$

So far, we have one equation and two unknowns, m_2 and T_2 . We increase the number of equations by using the equation of state for state 2:

$$P_2 V = m_2 R_u T_2 / M_{air}$$

where $V_2 = V_1 = V$. Substituting for $m_2 T_2$ from the equation of state in the energy equation we find m_2 :

$$m_2 = \frac{P_2 V c_v (M_{air} / R_u) + m_1 (h_i - c_v T_1)}{h_i} \quad \text{IIa.8.4}$$

Total mass entering the tank is therefore $m_i = m_2 - m_1$ and the average mass flow rate is found from

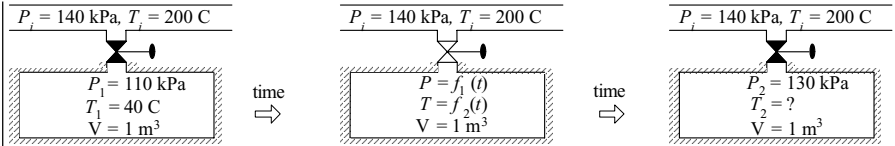
$$\bar{m}_i = m_i / \tau$$

where τ is the time it took to bring the tank pressure from P_1 to P_2 . By eliminating m_2 between the energy equation and the equation of state, we can also obtain an expression for T_2 as:

$$T_2 = \left[\left(1 - \frac{P_1}{P_2}\right) \frac{1}{\gamma T_1} + \frac{P_1}{P_2} \frac{1}{T_1} \right]^{-1} \quad \text{IIa.8.5}$$

where we have substituted for $h_i = c_p T_i$ and then $\gamma = c_p / c_v$.

Example II.8.3. A well-insulated tank contains air at 110 kPa and 40 C with the isolation valve closed. We now open the valve and let pressurized air enter the tank. We close the valve in 6 seconds when pressure in the tank reaches 130 kPa. Find the amount of air that has entered the tank. $M_{air} = 28.97 \text{ kg/kmol}$.



Solution: $P_1 = 110 \text{ kPa}$, $T_1 = 40 \text{ C}$ (313 K), $V = 1 \text{ m}^3$. Initial mass of air in the tank is:

$$m_1 = P_1 V / RT_1 = 110 \times 1 / [(8.314/28.97) \times 313] = 1.22 \text{ kg}$$

We find h_i , assuming constant c_p :

$$h_i - h_{ref} = c_p(T_i - T_{ref}) = 1.0(200 + 273 - 0) = 473 \text{ kJ/kg (note } h_{ref} = 0 \text{ at } T_{ref} = 0 \text{ K)}.$$

To find m_2 , we may use Equation IIa.8.4 (or find T_2 from Equation IIa.8.5 then m_2 from $m_2 = P_2 V / RT_2$):

$$m_2 = \frac{[130 \times 1 \times 0.72 \times (28.97 / 8.314)] + 1.22[473 - 0.72 \times (40 + 273)]}{473} = 1.33 \text{ kg}$$

The mass of air entering the tank is therefore $1.33 - 1.22 = 0.11 \text{ kg}$. To find the average flow rate, we divided $m_2 - m_1$ by the charging duration: $0.11/6 \cong 0.02 \text{ kg/s}$. To find T_2 , we use the equation of state:

$$T_2 = P_2 V / m_2 R = 340 \text{ K} = 67.6 \text{ C}.$$

In Example IIa.8.3, we were able to find an analytical solution in a closed form because the working fluid could be treated as an ideal gas, allowing the use of a simple equation of state. The reader may try the above example with steam being the working fluid. In the first try, assume that the container is initially evacuated. In the second try, assume that the container has steam at atmospheric pressure and 121 C (250 F).

Case B. For Given \dot{m}_i , Find P_2 and T_2 :

Given the initial conditions, our goal is to find the final pressure and temperature of a rigid vessel versus time while the vessel is being charged with an ideal gas at a specified mass flow rate and enthalpy. We follow the same procedure as in Case A and this time solve the equation for T_2 :

$$T_2 = (m_1 u_1 + m_i h_i) / (m_1 + m_i) c_v$$

Having T_2 , and m_2 , we can find P_2 .

Example IIa.8.4. A pressure vessel has a volume of 100 ft^3 (2.83 m^3). It contains air at 1000 psia ($\sim 7 \text{ MPa}$) and 150 F (65.5 C). A valve is now opened and highly pressurized air at a rate of 1 lbm/s (0.453 kg/s) and a temperature of 292 F (144.4 C) enters the vessel. Determine the gas pressure and temperature in the vessel after 1 minute of charging. $R_{air} = R_u / M_{air} = 1545/28.97 = 53.33 \text{ ft}\cdot\text{lb}/\text{lbm}\cdot\text{R}$.

Solution: $m_1 = P_1 V / R_{air} T = (1000 \times 144) \times 100 / (53.33 \times 610) = 442.9 \text{ lbm (200 kg)}$

$$T_2 = [442.9 \times 0.171 \times 610 + (1 \times 60) \times 0.24 \times 752] / [(442.9 + 1 \times 60) \times 0.171] = 663 \text{ R} = 203 \text{ F (95 C)}.$$

$$m_2 = m_1 + \bar{m} \Delta t = 442.9 + 1 \times 60 = 502.9 \text{ lbm (228 kg)}$$

$$P_2 = m_2 R_{air} T / V = 502.9 \times 53.33 \times 663 / 100 = 1235 \text{ psia (8.5 MPa)}.$$

8.3. Charging Vessels with Gas (Expanding C.V.)

A cylinder equipped with a frictionless piston contains air at pressure P_1 and temperature of T_1 . Our goal is to find the mass of the air entering the cylinder when the air temperature at the final state reaches T_2 . As shown in Figure IIa.8.3(a), at the initial state (P_1, T_1) the intake valve is closed. In Figure IIa.8.3(b), the intake valve has opened, allowing air to enter the control volume. Figure IIa.8.3(c) shows the final state where the intake valve is again closed and the air temperature has reached T_2 . Since the piston is allowed to move, the pressure of the air in the cylinder remains at P_1 throughout the filling process. The moving boundary also requires accounting for the work performed by the piston moving against the atmospheric pressure.

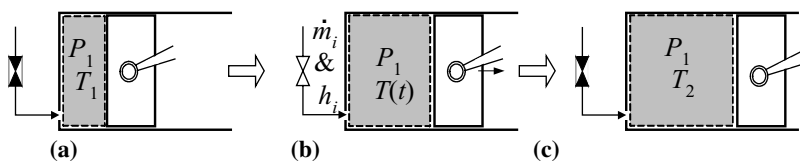


Figure IIa.8.3. Charging expanding control volumes: (a) initial state, (b) filling process, and (c) final state

We now integrate the conservation equations for mass and energy from state 1 to state 2. From the continuity equation, the mass of gas at the final stage is given as:

$$m_2 = m_1 + m_i$$

and from the conservation equation of energy we conclude that:

$$m_1 u_1 + m_i h_i + Q = P(V_2 - V_1) + m_2 u_2$$

We have two equations and three unknowns, m_i , m_2 , and u_2 for specified m_1 , u_1 , h_i , Q , P , V_1 , and V_2 . Treating air as an ideal gas, the equation of state becomes $PV_2 = m_2 RT_2$, which also satisfies the volume constraint ($v_2 = V_2/m_2$). We now solve for m_i :

$$m_i = [c_v P V_2 / R - m_1 u_1 - Q + P(V_2 - V_1)] / h_i \quad \text{IIa.8.6}$$

Example IIa.8.5. A cylinder has an initial volume of 1.0 ft^3 and contains air at 15 psia (0.1 MPa) and 120 F (49 C). Air at 150 psia (1 MPa) and 250 F (121 C) is injected into the cylinder, pushing the piston to a new position where $V_2 = 3V_1$. In the process, 0.2 Btu (211 J) heat is transferred from the cylinder to the surroundings. Find the mass of air entering the cylinder and the final air temperature in the cylinder. $R_{\text{air}} = R_u/M = 1545/28.97 = 53.33 \text{ ft}\cdot\text{lbf/lbm}\cdot\text{R}$ and $c_v = 0.171 \text{ Btu/lbm}\cdot\text{R}$.

Solution: $m_1 = PV_1/RT_1 = (15 \times 144) \times 1/[53.33(460 + 120)] = 0.07 \text{ lbm}$
 $m_i = [0.171 \times (15 \times 144) \times 3/53.33 - 0.07 \times 0.171 \times 580 - (-0.2) + 15 \times 144$
 $(3 - 1)/778]/(0.24 \times 710)$

$m_i = 0.115 \text{ lbm}$ (0.05 kg)

$T_2 = 15 \times 144 \times 3/[(0.07 + 0.115) \times 53.33] = 657 \text{ R} = 197 \text{ F}$ (92 C).

8.4. Discharging Gas-Filled Rigid Vessels (Fixed C.V.)

Determination of the rate of depressurization of vessels filled with fluids under pressure poses a challenging task, especially when the vessel is filled with liquid. Depending on fluid pressure and the rate of discharge, the liquid may change phase and flash to vapor. Vessel depressurization and flow of two-phase mixture through pipes are discussed in Chapter Va. Here we deal with an easier task of analyzing the depressurization of rigid vessels filled with an ideal gas in an isentropic process. For non-isentropic processes, see the accompanying CD-ROM.

The vessel is the control volume. In this ideal process there is no heat transfer and we assume that the depressurization process is reversible. Therefore, from Equation IIa.4.4 we have $Tv^{\gamma} = \text{constant}$. Also the volume constraint requires that $V = mv = \text{constant}$. Taking the derivative of the isentropic relation, we find:

$$\frac{dT}{T} = (1 - \gamma) \frac{dv}{v} \quad \text{IIa.8.7}$$

Similarly, the derivative of the volume constraint yields:

$$\frac{dm}{m} = - \frac{dv}{v} \quad \text{IIa.8.8}$$

Solving these equations simultaneously, we find that at any point in time we have:

$$\frac{m_2}{m_1} = \left(\frac{T_2}{T_1} \right)^{1/(\gamma-1)} \quad \text{IIa.8.9}$$

Substituting from Equation IIa.4.5 into Equation IIa.8.9 yields:

$$\frac{m_2}{m_1} = \left(\frac{P_2}{P_1} \right)^{1/\gamma} \quad \text{IIa.8.10}$$

We may also integrate Equation IIa.8.8 to obtain a relationship between mass and specific volume or between mass and density.

Example IIa.8.6. A tank is filled with air. A valve is opened to vent the tank. If pressure drops to 1/3 of its initial value find a) the mass of the gas left in the tank and b) final air temperature. The process is isentropic. Data: $V = 2 \text{ m}^3$, $P_1 = 6 \text{ bar}$, $T_1 = 230 \text{ C}$. Equilibrium within the tank during the process.

Solution: Treating air as an ideal gas, the initial mass of air is found as:

$$m_1 = \frac{P_1 V}{RT_1} = \frac{6 \times 2}{(0.08314/28.97)(230 + 273)} = 8.313 \text{ kg}$$

a) For the isentropic process $m_2 = \left(\frac{P_2}{P_1}\right)^{1/\gamma} m_1 = 8.313 \times (1/3)^{1/1.4} = 3.79 \text{ kg}$

b) The air temperature drops to: $T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} T_1 = \left(\frac{1}{3}\right)^{0.286} \times 503 = 367 \text{ K} = 94 \text{ C}$

Rapid discharge of pressurized vessels induces thermal stresses in the vessel wall

8.5. Dynamics of Gas Filled Vessels

Earlier we derived the mass and enthalpy of mixing tanks containing a liquid at constant pressure. We want to extend the derivation to vessels containing a gas. We consider a general case of simultaneous charging of the vessel with the same gas at several inlet ports and discharging the vessel while heat and shaft work are added to the vessel as shown in Figure IIa.8.4. The derivation in this case is mathematically more involved since the gas pressure in the tank changes with time. Like before, the simplifying assumptions include a) negligible changes in the *K.E.* and *P.E.*, b) instantaneous and perfect mixing of the incoming streams with the gas in the tank, c) no chemical reaction takes place in the tank throughout the process, and d) no heat loss from the tank to the surroundings. Expanding the time derivative term in Equation IIa.6.4-1, using the perfect mixing assumption ($h_e = h_{C.V.}$) and substituting from Equation IIa.5.1, we obtain:

$$m \frac{dh}{dt} = \sum \dot{m}_i (h_i - h) + \sum \dot{Q} - \sum \dot{W}_s + V \frac{dP}{dt} \quad \text{IIa.8.11}$$

Since we already used the continuity and the energy equations and have more unknowns than equations, we now take advantage of the volume constraint; $V_{C.V.} = (mv)_{C.V.} = \text{constant}$ or alternatively $dV_{C.V.}/dt = 0$:

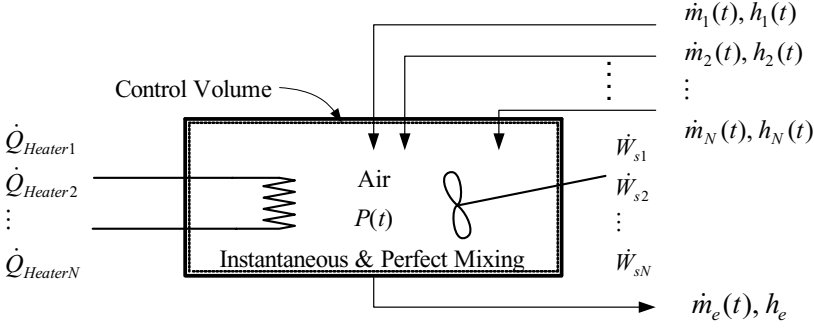


Figure IIa.8.4. A mixing tank containing an ideal gas

$$\frac{d(mv)_{C.V.}}{dt} = \frac{dm}{dt}v + m \frac{dv}{dt} = 0 \quad \text{IIa.8.12}$$

We now drop the subscript C.V., substitute for dm/dt from Equation IIa.5.1, and ponder what to do with the dv/dt term. Since P and h are the state variables, we expand dv/dt in terms of P and h , using the chain rule for composit functions:

$$\frac{dv}{dt} = \frac{\partial v}{\partial h} \frac{dh}{dt} + \frac{\partial v}{\partial P} \frac{dP}{dt} \quad \text{IIa.8.13}$$

Having all the ingredients, we proceed to substitute for dv/dt from Equation IIa.8.13 and for dh/dt from Equation IIa.8.11 into Equation IIa.8.12. We then rearrange the resulting equation and solve for dP/dt :

$$\frac{dP}{dt} = - \frac{v(\sum \dot{m}_i - \sum \dot{m}_e) + [\sum \dot{m}_i (h_i - h) + \sum \dot{Q} + \sum \dot{W}_s] (\partial v / \partial h)}{m(\partial v / \partial P) + V(\partial v / \partial P)} \quad \text{IIa.8.14}$$

At the first glance, Equation IIa.8.14 appears intimidating especially since we have introduced such unfamiliar terms as $\partial v / \partial h$ and $\partial v / \partial P$. However, this equation can be easily solved by finite difference, for example. As for the partial derivative terms, the equation of state comes to our rescue. If we are dealing with ideal gases $Pv = RT$ and therefore, $v = RT/P$. We also have $dh = c_p dT$ so that;

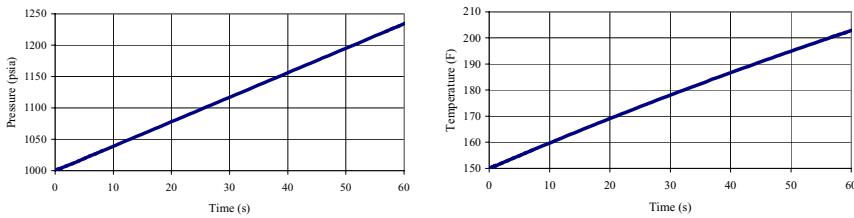
$$\frac{\partial v}{\partial P} = -\frac{v}{P} \quad \text{and} \quad \frac{\partial v}{\partial h} = \frac{\partial v}{\partial T} \frac{\partial T}{\partial h} = \frac{1}{c_p} \frac{R}{P} \quad \text{IIa.8.15}$$

We can now find dh/dt by substituting for dP/dt from Equation IIa.8.14 into Equation IIa.8.11. Pressure and enthalpy of the gas in the rigid vessel can then be calculated by subsequent integration.

In this derivation, we considered only rigid vessels thus, a control volume with fixed boundary. Control volumes with moving boundaries are analyzed in Chapter VI d.

Example Ila.8.7. Solve Example Ila.8.4 using Equations Ila.8.11 and II.8.14.

Solution: Equations Ila.8.11 and Ila.8.14 are non-linear differential equations, which we solve by the finite difference method. The solution by FORTRAN is included on the accompanying CD-ROM. The results are shown in the plots of P and T versus time. Pressure and temperature in 1 minute reach 1235 psia (8.5 MPa) and 203 F (95 C), respectively.



Special Case; Isothermal Process

Consider the rigid tank of Figure Ila.8.5. The tank is initially at T_1 and $P_1 > P_{atm}$. We open a small valve and vent the tank while simultaneously adding heat to the tank to maintain the air temperature in the tank at its initial value. We want to determine the amount of heat added to the tank when the pressure drops to P_2 .

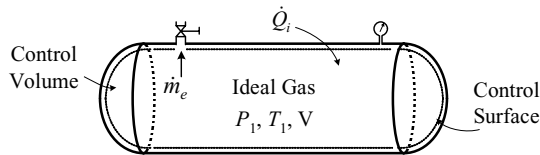


Figure Ila.8.5. Discharging gas-filled rigid vessels

While we can solve this problem by using Equations Ila.8.11 and II.8.14, we instead choose the direct solution by using Equations Ila.5.1 and Ila.6.4 in addition to the equation of state. Since there is no inlet stream, no shaft work, and only one exit port, Equation Ila.6.3 is simplified to:

$$\frac{d(mu)_{C.V.}}{dt} = m_{C.V.} \frac{du}{dt} + u_{C.V.} \frac{dm_{C.V.}}{dt} = -\dot{m}_e h_e + \dot{Q}$$

The logic for setting the term involving du/dt equal to zero is as follows: $du/dt = d(c_v T)/dt$. If we assume c_v remains constant then $du/dt = c_v(dT/dt)$. Since the proc-

process takes place at constant temperature then $dT/dt = 0$ thus, $du/dt = 0$. We now substitute from the continuity equation, $dm/dt = -\dot{m}_e$, to obtain:

$$u_{C.V.}(dm_{C.V.}/dt) = h(dm_{C.V.}/dt) + \dot{Q}$$

where due to the perfect mixing assumption, we also substituted for $h_e \equiv h$. Rearranging this equation yields:

$$\dot{Q} = (u - h)(dm/dt) = [u - (u + Pv)](dm/dt) = (Pv)(dm/dt) = RT(dm/dt)$$

where subscript C.V. is dropped. The amount of heat added to the tank is found by integrating this equation:

$$Q_{1-2} = RT \int_1^2 dm = RT(m_2 - m_1) = V(P_2 - P_1)$$

Example IIa.8.8. A 0.5 m^3 rigid tank is filled with air at 38 bar and 65 C. A valve is opened to slowly vent the tank. Find the amount of heat addition to the tank so that temperature remains at 65 C while pressure drops to 1 bar.

Solution: Treating air as an ideal gas, we find

$$Q_{1-2} = 0.5 \times (1 - 38) \times 1\text{E}5 = -1850 \text{ kJ}.$$

As an exercise, solve this problem by using Equations IIa.8.11 and II.8.14.

Calculation of P and h from Equation IIa.8.11 and II.8.14 requires specification of such input data as heater power, shaft work, inlet enthalpy and the mass flow rates at inlet and exit ports. Regarding mass flow rate at the exit port, if the control volume mass at state 2 (i.e., m_2) is specified then \dot{m}_e can be found from Equation IIa.8.1. Otherwise, \dot{m}_e is a function of tank pressure and temperature, $\dot{m}_e = f[P(t), T(t)]$ and we must calculate the mass flow rate at the exit port from an additional equation. This additional equation is the momentum equation written between the valve inlet and outlet ports. We leave further discussion of this topic to Chapter IIIc.

8.6. Discharging Rigid Vessels (Fixed C.V.) Filled with Two-Phase Mixture

In Section IIa.7.8, we examined cases where heat was added to the two-phase mixture in a control volume but no mass was allowed to enter or leave the control volume. Here we study the case of letting mass leave the control volume. Initially, the vessel contains a saturated mixture of water and steam at equilibrium (state 1 in Figure IIa.8.6). Adding heat to a rigid vessel in an isobaric process requires mass to be withdrawn. In this special case, we remove only saturated steam through a vent valve at the top of the vessel. We stop adding heat to the vessel and removing steam from the vessel when the last drop of water becomes satu-

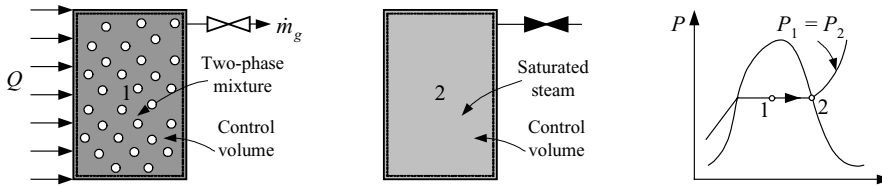


Figure Ila.8.6. Discharging steam and adding heat to a vessel at constant pressure

rated steam (state 2 in Figure Ila.8.6). We want to find the amount of heat needed for this process.

The solution to this problem is obtained from Equation Ila.8.11. However, for this isobaric process we can find an analytical solution in closed form. In this process, a carefully controlled heat addition and steam removal maintains the vessel pressure at its initial state throughout the isobaric vaporization process.

Since steam leaves the vessel at constant pressure, $\dot{m}_e = \dot{m}_g$ and $h_e = h_g$. The conservation equation for energy then becomes:

$$\dot{Q} = \dot{m}_g h_g + d(mu)_{CV} / dt$$

Multiplying both sides by dt and integrating gives;

$$\int \dot{Q} dt = h_g \int \dot{m}_g dt + (m_2 u_g - m_1 u_1).$$

We now substitute from the continuity equation to obtain:

$$Q_{12} = (m_1 - m_2) h_g + (m_2 u_g - m_1 u_1)$$

We calculate m_1 and m_2 from the equation of state and the volume constraint, $m_1 = V/v_1$ and $m_2 = V/v_g$.

Example Ila.8.9. A tank having a volume of 40 m^3 contains a mixture of water and steam at 7 MPa and a steam quality of 0.65 . Steam is withdrawn from the top of the tank while heat is added in an isobaric process until the steam quality becomes 100% . Find the amount of heat added to the tank and the mass withdrawn.

Solution:

P (MPa)	v_f (m^3/kg)	v_g (m^3/kg)	u_f (kJ/kg)	u_g (kJ/kg)	h_g (kJ/kg)
7	1.108E-3	0.2729	696.44	2572.5	2763.5

$$v_1 = 1.108\text{E-}3 + 0.65 \times 0.2718 = 0.178 \text{ m}^3/\text{kg}$$

$$m_1 = 40/0.178 = 225 \text{ kg}$$

$$m_2 = 40/0.2729 = 146.6 \text{ kg}$$

$$m_e = 225 - 146.6 = 78 \text{ kg}$$

$$u_1 = 696.44 + 0.65 \times (2572.5 - 696.44) = 1915.88 \text{ kJ/kg}$$

$$Q_{12} = (m_1 - m_2)h_g + (m_2u_g - m_1u_1) = 78 \times 2763.5 + (146.6 \times 2572.5 - 225 \times 1915.88) = 162.7 \text{ MJ}$$

8.7. Pressure Search for a Control Volume

As was discussed in Example IIa.3.4, often we calculate v and u for a control volume from which we need to find the control volume pressure and temperature. This requires a solution based on iteration with the steam tables. Let's consider a case where the final state is a saturated mixture. In this case, we may substitute for quality from $v = v_f + x v_{fg}$ into $u = u_f + x u_{fg}$ and obtain the following relation:

$$(u - u_f)v_{fg} + (v - v_f)u_{fg} = 0$$

If v_f , v_{fg} , u_f , and u_{fg} are now expressed as functions of either pressure or temperature, we can solve for pressure (or temperature) using the Newton-Raphson method, as discussed in Chapter VIIe. Having found pressure (or temperature), the corresponding saturation temperature (or pressure) can then be found. Shown in Table A.II.3 are examples of curves fits to data for v_f , v_{fg} , u_f , and u_{fg} in terms of T .

Example IIa.8.10. A tank having a volume of 500 ft³ contains a homogenous mixture of water and steam at 400 F. The initial steam quality is 15%. We now add 200 lbm of water at 450 psia and 350 F to the tank. Find pressure and temperature, assuming perfect mixing of water with the mixture in the vessel.

Solution: We follow the steps outlined below:

T (F)	P (psia)	v_f (ft ³ /lbm)	v_g (ft ³ /lbm)	u_f (Btu/lbm)	u_g (Btu/lbm)
400	247.26	0.01864	1.8630	372.45	1115.74

$$v_1 = 0.01864 + 0.15 \times (1.8630 - 0.01864) = 0.2953 \text{ ft}^3/\text{lbm}$$

$$m_1 = 500/0.2953 = 1693.23 \text{ lbm}$$

$$u_1 = 372.45 + 0.15 \times 743.29 = 483.94 \text{ Btu/lbm}$$

$$m_2 = m_1 + m_{add} = 1893.23 \text{ lbm.}$$

The enthalpy of the added water: $h_{add}(450 \text{ psia} \ \& \ 350 \text{ F}) = 322.24 \text{ Btu/lbm.}$

Applying Equation IIa.6.4 to the control volume representing the tank gives:

$\dot{m}_i h_i = d(mu) / dt$. Integrating and solving for u_2 , we obtain:

$$u_2 = \frac{m_1 u_1 + m_{add} h_{add}}{m_1 + m_{add}}$$

We also have $v_2 = V/(m_1 + m_{add})$. The numerical values for u_2 and v_2 are calculated as:

$$u_2 = [1693.23 \times 483.94 + 200 \times 322.24]/1893.23 = 466.86 \text{ Btu/lbm.}$$

$$v_2 = 500/1893.23 = 0.264 \text{ ft}^3/\text{lbm}$$

Having u_2 and v_2 , we find P_2 and T_2 by iteration with the steam tables for saturated mixture as $P_2 = 239.95$ psia, $T_2 = 297.4$ F, and $x_2 = 0.128$. Expectedly, adding colder water reduces the mixture enthalpy.

9. The Second Law of Thermodynamics

In the previous sections dealing with the first law of thermodynamics, we stated that both heat and work are forms of energy. We also showed the relationship between heat and work. There were several observations that were missing in those discussions. For example, we have noted from experience that work can be readily transformed to heat whereas the reverse is not readily possible. Furthermore, while 100% of work can be transformed to heat, conversion of heat to work is always less efficient. Another important fact is the effect of temperature on storage of thermal energy (i.e., the higher the temperature of the stored thermal energy, the higher the ability to be converted into work). Perhaps the most interesting observation regarding energy conversion is bringing a hot block of steel in contact with a colder block of steel. Intuitively, we know the heat flows from the warmer to the colder block. However, there is no provision in the first law to prohibit the flow of heat from the colder to the warmer block. The first law is concerned only with the conservation of energy in a process and not with the direction of the process. It is the second law that establishes the possible direction of a process. Another example includes the daily dumping of vast amounts of energy to the surroundings at power plants where work is produced in the form of electricity. Production of work equal to the same amount of energy delivered to the heat source is not prohibited by the first law. However, the loss of energy to the surroundings (i.e., the requirement for a heat sink) can be explained only if put in the framework of the second law of thermodynamics. As was stated in Chapter I, unlike the first law, the second law is a not a conservation law.

9.1. Definition of Terms

Work and heat reservoirs are two thermodynamic concepts. A work reservoir is a system for which every unit of energy crossing its boundary is in the form of work. Examples of a work reservoir include a perfectly insulated turbine and a perfectly elastic compressed spring. The heat reservoir is a constant temperature body as heat is transferred into or out of the body. A large lake acting as a heat sink for a power plant may be considered as a heat reservoir. Comparing two heat reservoirs at two different temperatures, the heat reservoir at the higher temperature is referred to as the heat source and the heat reservoir at the lower temperature, the heat sink.

Heat source is referred to any hot heat reservoir. In a gasoline engine, the heat source is the combustion chamber at the moment that the compressed gases are ignited and burn due to the action of a spark plug. In a jet engine or gas turbine power plant, the heat source is the combustion chamber where compressed air enters to mix with the injected fuel for combustion. In a fossil plant, the heat source is the boiler. In a BWR, the heat source is the reactor vessel and in a PWR, the heat source is the secondary side of the steam generator.

Heat Sink refers to any cold heat reservoir. In a gasoline engine, the heat sink is the radiator. In a power plant located next to a large body of water, the heat sink is the condenser. Power plants not having access to large bodies of water use cooling towers as heat sinks. In a heated room with no windows, the heat sink consists of the ceiling, the floor, and the walls. If an air conditioning unit is now installed to cool this room and we assume the walls quickly reach thermal equilibrium with the room, the primary heat sink for the room is the air conditioning unit. This is however, an intermediate heat sink as eventually heat is transferred to the surroundings. As a result, the environment is the *ultimate heat sink*.

Cycle is a process that, after completion, brings the system to its original state. As a result, the net change in any property of the system is zero. As an example, consider the motion of piston in cylinder of Figure IIa.9.1. We may start from a point where the piston is fully inserted and gas is at the highest pressure. The first process or path includes the expansion of the gas, which forces the piston to the bottom of the cylinder. This also turns the flywheel. The second process is when the stored energy in the flywheel pushes the piston back to its original position completing one cycle.

Clausius statement of the second law deals with the transfer of energy from a heat sink to a heat source. Simply stated, the Clausius statement specifies that “it is impossible for any device to operate in a cycle and produce no effect other than the transfer of energy by heat from the heat sink to the heat source.” In other words, the Clausius statement clarifies that the operation of heat pumps and refrigerators is possible only if work is provided to the device (compressor) to accomplish the task of removing heat from a heat sink and transferring it to the heat source.

Kelvin-Planck statement of the second law deals with the transfer of energy from a heat source to a heat sink. This statement specifies that “it is impossible for any device to operate in a cycle and produce work with only a heat source.” In other words, the Kelvin-Planck statement clarifies that no power plant can operate with a boiler, an engine, or a combustion chamber but without a radiator, a cooling tower, or a condenser.

Reversible process as defined earlier refers to a process that, if applied to a system, can be reversed exactly to the initial state with no change in the system or its surroundings. A reversible process is hard to achieve and can only be approached in a carefully planned and executed process. Examples of processes that can approach a reversible process include a smooth converging-diverging nozzle.

Among mechanical systems that are equipped with a flywheel and have the potential of approaching a reversible process we may consider the periodic motion of a pendulum in a vacuum container with negligible friction at the base. Similarly, as shown in Figure IIa.9.1, the operation of a frictionless well-insulated piston in the well-insulated cylinder while attached to a flywheel approaches a reversible process.

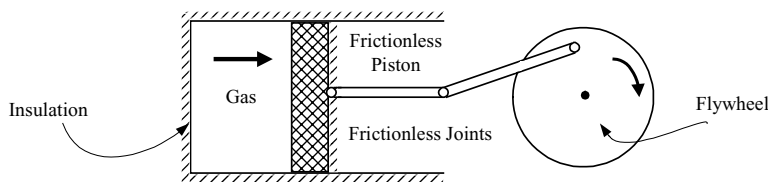


Figure IIa.9.1. A frictionless, well insulated system approaching reversible process

To illustrate how a process can be made reversible, consider the frictionless piston in Figure IIa.9.2 fixed in place by a pin. Pressure inside the cylinder is P_1 . We now release the pin and the piston reaches the stops at pressure P_2 . This process is not reversible, because during the expansion, the piston pushes against atmospheric pressure. The force needed to push the piston back to its original place is larger as the piston has to push against $P_2 > P_{atm}$. This results in work to be delivered to the piston. To approach a reversible process, consider the same piston but now it is attached to a linear spring with $k_{spring} = P_1 A/L$.

It is important to remember that there are no dissipative effects upon the conclusion of a reversible process.

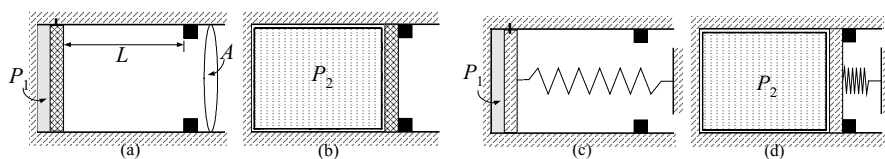


Figure IIa.9.2. Transformation of a system to approach reversible process

Irreversible process refers to any process, which is not reversible. In practice, all engineering processes are irreversible. Friction in the form of heat loss to the surroundings is one of the main reasons for this irreversibility. For example, as discussed in Chapter IIIb, the flow of fluids in pipelines and in the bends of conduits is always associated with unrecoverable pressure loss. This is due to the fluid shear stresses and roughness of the pipe wall. There are, of course, other types of irreversible processes such as shock waves resulting in sonic booms and any hysteresis effect. Inelastic deformation where a solid does not return to its original dimensions following removal of the applied force is an irreversible process. Flow of electric current through an electric resistance produces heat, causing

the process to be irreversible. Spontaneous mixing of substances of different compositions, and all actual heat transfer mechanisms are also examples of irreversible processes. The latter is an irreversible process as to reverse the process, a refrigeration cycle is needed to transfer heat from the heat sink to the heat source. This requires transfer of work from the surroundings. In practice, we can reduce certain irreversibilities by taking such actions as using smooth piping for internal flow, contoured or streamlined surfaces for external flow, and lubrication for solid to solid contact. There are always dissipative effects upon the conclusion of the irreversible process. Hence, in the design and operation of systems we must focus on reducing the irreversibilities associated with a system to minimize their dissipative effects and maximize efficiency.

Internal and external irreversibilities are two categories of irreversible processes with respect to the system boundary. Internal irreversibilities occur inside the boundary of a system and are associated with friction due to fluid shear stresses and such other processes as fluid expansion as well as fluid mixing. External irreversibilities occur across the system boundary and are associated with heat transfer to or from the system, friction due to the mechanical motion such as shaft rotation in bearing, and windage losses in electric generators.

Reversible work, W_{rev} is the work done by or on a system when it undergoes a reversible process.

Irreversible work, W_{irr} is the work done by or on a system when it undergoes an irreversible process. In a work producing system undergoing different paths, all beginning and ending in an identical change of state, the reversible work produced by the reversible path is the maximum work that can be obtained. Similarly, for a work absorbing system undergoing different paths, all beginning and ending with an identical change of state, the reversible work absorbed in the reversible path is the minimum work that can be absorbed.

Irreversibility, $I = W_{rev} - W_{irr}$ is the difference between the reversible and the irreversible work for a system when it undergoes reversible and irreversible cycles beginning and ending in an identical change of state. Since the reversible work is always larger than the irreversible work for work producing systems, and always smaller than the irreversible work for work absorbing systems, the irreversibility I is always a positive quantity. The irreversibility, also referred to as the *lost work*, is discussed further in the next section.

Heat engine is a work reservoir that goes through a cycle to produce work while heat is being transferred to and from the system across its boundary. As shown in Figure IIa.9.3, heat is transferred to the heat engine from the heat source and is transferred from the engine to the heat sink. Work is produced in this process.

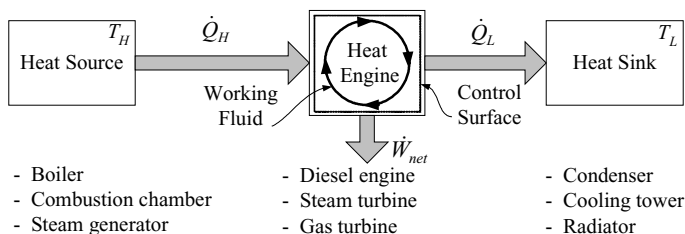


Figure IIa.9.3. Schematic of a heat engine in steady state operation

Thermal efficiency for a heat engine is defined as the net energy output in steady state operation from the engine in the form of work divided by the energy input to the heat engine from the heat source. Perhaps the most intuitive definition of efficiency is the ratio of energy obtained to energy spent. Using our sign convention (i.e., plus sign for heat transferred to the system and work delivered by the system and minus sign for heat transferred from the system and work transferred to the system) the first law for steady state operation becomes:

$$+(\dot{Q}_H) - (\dot{Q}_L) = +(\dot{W}_{net})$$

Thermal efficiency becomes:

$$\eta_{th} = \frac{\text{energy obtained}}{\text{energy spent}} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} \quad \text{IIa.9.1}$$

Equation IIa.9.1, despite its simplicity, conveys important information. For example, according to the second law, $|\dot{Q}_L|$ is always greater than zero. As such, thermal efficiency of a heat engine can never be 100%. In the remainder of this chapter, we will see that thermal efficiency of a heat engine is indeed much smaller than unity. Equation IIa.9.1 also shows that to increase thermal efficiency for a given rate of heat transfer from the heat source, we must reduce the rate of heat transfer to the heat sink.

Carnot principle states that a reversible heat engine always has a higher thermal efficiency than an irreversible heat engine. The Carnot principle (Nicolas Leonard Sadi Carnot, 1796 - 1832) also states that two reversible heat engines operating between identical heat sources and heat sinks have identical thermal efficiencies.

Kelvin temperature scale provides a simple relation between the ratio of heat transfers to the heat sink and the heat source versus the temperature of these reservoirs. Referring to Figure IIa.9.3, in general the ratio of the rate of heat transfers can be expressed by several functions. Kelvin (William Thomson later became Lord Kelvin, 1824 - 1907) suggested:

$$\frac{\dot{Q}_L}{\dot{Q}_H} = f(T_L, T_H) = \frac{T_L}{T_H}$$

Carnot efficiency is derived from the Carnot principle, correlating thermal efficiency of reversible heat engines solely to the heat source and the heat sink temperatures. According to Kelvin's suggestion for a temperature scale, Equation IIa.8.3 becomes (see derivation in Chapter IIb):

$$\eta_{th,Carnot} = 1 - \frac{T_L}{T_H} \quad \text{IIa.9.2}$$

where T_L and T_H are absolute temperatures. Equation IIa.9.2 is the Carnot thermal efficiency for heat engines. This simple, yet very important equation expresses that no heat engine can have a thermal efficiency higher than that predicted by Equation IIa.9.2. Also note that the higher the temperature of the heat source, the higher the thermal efficiency. However, achievement of high temperatures in practice is limited to the metallurgical characteristics of the materials constituting the heat engine.

Example IIa.9.1. Steam pressure in the secondary side of a PWR steam generator is 900 psia (6.2 MPa). The condenser uses bay water, the lowest temperature of which is 40 F (4.4 C). Determine the maximum thermal efficiency this plant could achieve.

Solution: From Equation IIa.9.2

$$(\eta_{th})_{Max} = 1 - \frac{40 + 460}{T_{sat}(900) + 460} = 1 - \frac{40 + 460}{531.95 + 460} = 49.6\%$$

Due to irreversibilities, power plants using a steam cycle have thermal efficiency of about 30%.

Thermal pollution refers to the adverse environmental impact that power plants could have on the surroundings as the ultimate heat sink. The warm water at the exit of a once-through condenser has a temperature ranging from 12 to 25 F above the temperature of the water at the inlet. The effect of this temperature rise on the ecosystem depends on the size of the body of water ranging from a river or a lake to an estuary or an ocean.

Example IIa.9.2. An electric utility plans to operate a 1200 MWe power plant next to a lake. Agencies for protection of the environment have limited the rise in the lake water temperature to no more than 13 F (7 C). Determine the required flow of water to the condenser. Propose an alternative solution if this criterion cannot be met.

Solution: The percentage of a power plant's thermal efficiency ranges from high 20s to low 40s. Higher values of \dot{Q}_L is associated with lower thermal efficiency. Using a thermal efficient of 30% we find:

$$\dot{Q}_H = \dot{W}_{net} / \eta_{th} = 1200/0.3 = 4000 \text{ MW}$$

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{net} = 4000 - 1200 = 2800 \text{ MW}$$

This amount of energy is lost in the condenser to the environment. To find the required flow rate of cooling water to the condenser, we use an energy balance written between the inlet and outlet of the condenser, $\dot{Q}_L = \dot{m}c\Delta T$ where c is the specific heat of water. Its value between 20 C and 99.6 C is relatively constant at $c_{water} = 4.18 \text{ kJ/kg}\cdot\text{C}$. Using $\Delta T = 13 \text{ F}/1.8 = 7.2 \text{ C}$, the flow rate needed is therefore obtained from:

$$\dot{m} = 2.8\text{E}6 / [4.18 \times 7.2] = 93,000 \text{ kg/s} = 737\text{E}6 \text{ lbm/h} = 1.5\text{E}6 \text{ GPM} = 93 \text{ m}^3/\text{s}$$

This is a massive amount of water, which must be circulated through the condenser. If this flow rate cannot be sustained, the outlet temperature would exceed the limit. Cooling towers would assist in the task of removing heat as discussed in Chapter IIc.

In the above example, if we had used a thermal efficiency of 40%, which is an improvement of about 33%, the required flow rate would have dropped to 59,625 kg/s (59 m³/s). This is a reduction of about 36%, indicating that the reduction in the rate of heat loss to the surroundings, due to the increase in thermal efficiency, is greater than the increase in thermal efficiency itself. The effect of η_{th} on \dot{Q}_L and $\dot{Q}_L / \dot{W}_{net}$ for a $\dot{W}_{net} = 1000 \text{ MW}$ plant is shown in Figure IIa.9.4.

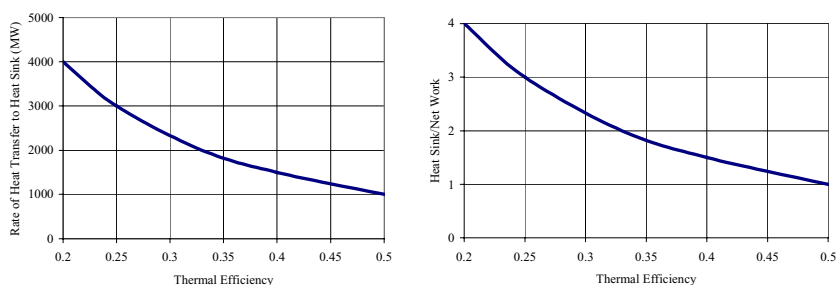


Figure IIa.9.4. Effect of Thermal efficiency on the rate of heat transfer to heat sink

Heat pump is a work reservoir that goes through a cycle and consumes work while heat is being transferred to and from the system across its boundary. As shown in Figure IIa.9.5, work is delivered to the heat pump to transfer heat from the heat sink to the heat source.

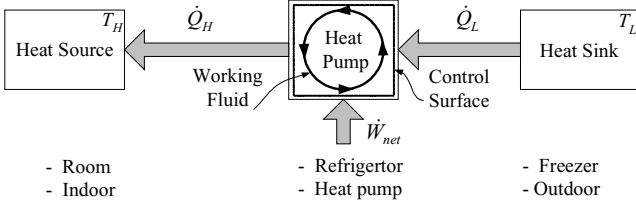


Figure IIa.9.5. Schematic of a heat pump in steady state operation

We have used the term “heat pump” as the reverse of a “heat engine”. These are both generic terms. While heat engine applies to such systems as an automobile engine, a steam turbine, and a jet engine, the heat pump applies to such systems as a refrigerator as well as a building heater/cooler. A refrigerator removes heat from the heat sink while a heat pump delivers heat to the heat source.

Coefficient of performance is a term defined for the refrigeration and heat pump cycles. In both cases, the coefficient of performance (COP) is defined similar to thermal efficiency for heat engines;

$$\text{COP} = \eta = \frac{\text{energy obtained}}{\text{energy spent}}$$

In refrigerators:

$$\eta_{\text{Refrigerator}} = \frac{\text{energy obtained}}{\text{energy spent}} = \frac{\dot{Q}_L}{\dot{W}_{\text{net}}} = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L}$$

and in heat pumps:

$$\eta_{\text{Heat pump}} = \frac{\text{energy obtained}}{\text{energy spent}} = \frac{\dot{Q}_H}{\dot{W}_{\text{net}}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L}$$

We now consider a reversible heat pump cycle. Such a cycle, according to the Carnot principle, consumes the least energy compared to an irreversible heat pump cycle. Using Lord Kelvin’s temperature scale for reversible cycles:

$$\left(\frac{\dot{Q}_L}{\dot{Q}_H}\right)_{\text{Reversible}} = \frac{T_L}{T_H} = r$$

The thermal efficiency and the COP for refrigerator and heat pump can be expressed as:

$$\eta_{\text{Heat engine, Carnot}} = 1 - r \quad \eta_{\text{Refrigerator, Carnot}} = \frac{r}{1 - r} \quad \eta_{\text{Heat pump, Carnot}} = \frac{1}{1 - r}$$

Example IIa.9.3. A heat pump is used for summer cooling and winter heating of a house. The heat pump COP is 5 and the rate of heat transfer to maintain the indoor temperature at 24 C when the outside temperature is 4 C is 5 kW. Find the power to operate the heat pump.

Solution: The power to operate the heat pump is obtained from $\text{COP} = 5/\dot{W}_{net}$. Therefore, $\dot{W}_{net} = 5/5 = 1$ kW. We may also find the maximum COP. If the heat pump was operating in a reversible cycle, the COP would have been $(\text{COP})_{\max} = 1/(1 - r)$ where $r = T_L/T_H = (4 + 273)/(24 + 273) = 0.93$ and $\text{COP} = 14.3$, indicating that the heat pump design could improved substantially to reduce the irreversibilities.

Carnot cycle for a heat engine results in the highest thermal efficiency of all power cycles. A cycle can be shown on pressure-volume (Pv) or temperature-entropy (Ts) coordinates. Consider the Carnot cycle, shown in the Ts diagram of Figure IIa.9.6. The first Ts diagram shows the Carnot cycle as an isentropic-isothermal cycle. Starting from Point 1, the working fluid is compressed isentropically to Point 2, which is at the temperature of the heat source. Heat is then transferred to the working fluid isothermally to Point 3 where the working fluid expands isentropically to produce work. Heat at Point 4 is then transferred isothermally until the cycle is completed at point 1. The cycle then would repeat. In the second Ts diagram, the area under the heat addition curve is shown to be equal to Q_H . In the third Ts diagram, the area under the heat rejection curve is shown to be Q_L . The net area is W_{net} . Hence:

$$W_{net} = Q_H - Q_L.$$

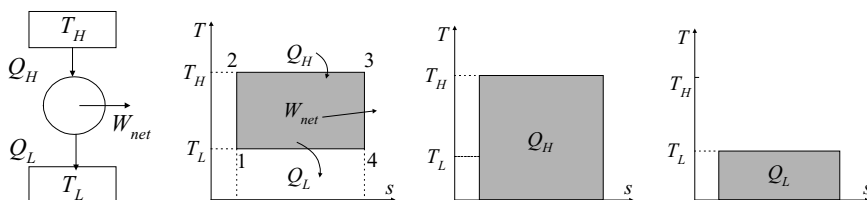


Figure IIa.9.6. Demonstration of a heat engine and Ts diagrams for the Carnot cycle

The Clausius inequality is expressed as:

$$\left[\oint \left(\frac{\delta Q}{T} \right)_{C.S.} = -\sigma \right] \leq 0$$

where the integral is taken over the control surface and the entire cycle. In this relation, σ is a measure of entropy production due to the existing irreversibilities in the system going through a cycle hence, σ is always positive for practical processes and can never be negative. The minimum value of σ is zero, occurring for only reversible processes. Later in this section we will show that σ is related to irreversibility (I) as $\sigma = I/T$.

Entropy, as a property of a system is the change in value of $\delta Q/T$ in a reversible process. We can then write:

$$S_2 - S_1 = \left(\int_1^2 \frac{\delta Q}{T} \right)_{rev} \quad \text{IIa.9.3}$$

Equation IIa.9.3 can also be written in differential form as $TdS = \delta Q$. If we now substitute for the right-hand side from Equation IIa.5.1, we get $TdS = \delta W + dE$. This can be simplified to:

$$TdS = dU + PdV \quad \text{IIa.9.4}$$

where only compression work in a reversible process is considered and the kinetic and potential energies are negligible. Entropy of a system may decrease, remain the same, or increase, depending on the process applied to the system. However, the net entropy of the system and its surroundings increases unless the process is reversible.

Exergy or availability determines the potential of a system to produce work. Any system can be at various levels of its availability. While availability is related to energy, unlike energy, availability is not conserved.

Power system refers to a heat engine that goes through a thermodynamic cycle to produce net work.

10. Entropy and the Second Law of Thermodynamics

Earlier we discussed the fact that energy is conserved and cannot be created or destroyed. We also learned about the first law of thermodynamics, which expresses the conservation of energy in various processes and noted that the first law does not provide any guideline for the direction of a process. It is the second law that clarifies the direction of a process. We also compared reversible with irreversible processes and noted that there are always dissipative effects associated with the irreversible processes. Such dissipative effects are evaluated in the context of availability versus *unavailability*. These terms are applied to the energy of a system. As such, the available energy is that amount of the energy of the system that can perform work. That portion of the energy of the system that cannot perform work is referred to as the unavailable energy. We can then write:

$$E_{\text{System}} = E_{\text{Available}} + E_{\text{Unavailable}} \quad \text{IIa.10.1}$$

Unlike energy, availability is not conserved. To elaborate consider an isolated system that includes fuel and air. Availability of this system prior to the combustion of the fuel is at its maximum as the fuel can be used to produce work. After combustion, the mixture of slightly warmer air and the combustion products has much less potential to perform work. Among various definitions for the entropy and the second law, entropy of a closed system can be defined as a property that is proportional to the unavailability of the system:

$$dS = C[dE_{\text{Unavailable}}] \quad \text{IIa.10.2}$$

where C in Equation IIa.10.2 is a proportionality constant. We can use Equation IIa.10.2 to readily show that the entropy change of a work reservoir is zero since, in a work reservoir, the unavailable energy is zero:

$$dS_{\text{Work reservoir}} = 0$$

In general however we can say that the unavailable energy is always positive or at least is equal to zero. Hence, we can write the second law for an isolated system as:

$$dS_{\text{Isolated system}} \geq 0 \quad \text{IIa.10.3}$$

Equation IIa.10.3 is the mathematical expression of the second law of thermodynamics for an isolated system and it describes the fact that the entropy of an isolated system can never decrease. Since we can consider any system and its surroundings as an isolated system, we can therefore write:

$$dS_{\text{System}} + dS_{\text{Surroundings}} \geq 0 \quad \text{IIa.10.4}$$

That is to say:

$$\text{For reversible processes:} \quad dS_{\text{System}} + dS_{\text{Surroundings}} = 0 \quad \text{IIa.10.4-1}$$

$$\text{For irreversible processes:} \quad dS_{\text{System}} + dS_{\text{Surroundings}} > 0 \quad \text{IIa.10.4-2}$$

Equation IIa.10.2 can also be used to determine the change in entropy for a heat reservoir. For this purpose, we first use the first law as given by Equation IIa.6.1 but expanded as:

$$\delta Q = \delta W + dE_{\text{Available}} + dE_{\text{Unavailable}} \quad \text{IIa.10.5}$$

Since for a heat reservoir, $dE_{\text{Available}}$ is only a fraction of δQ , Equation IIa.10.5 for a heat reservoir becomes:

$$\delta Q = C_1 dS$$

Rearranging in terms of dS , for a heat reservoir we obtain, $dS = \delta Q/C_1$. As shown by Hatsopoulos C_1 , the proportionality constant becomes $C_1 = 1/T$. Hence, for a heat reservoir:

$$dS_{\text{Heat reservoir}} = \frac{\delta Q}{T}$$

Since temperature of a heat reservoir remains constant, we can readily integrate the differential change in entropy to find that for a heat reservoir;

$$(S_2 - S_1)_{\text{Heat reservoir}} = \frac{Q_{12}}{T} \quad \text{IIa.10.6}$$

Using the sign convention, if the heat reservoir has gone through a process in which heat has been added to the reservoir, then $Q_{12} > 0$ and $S_2 - S_1 > 0$. On the other hand, if heat has been transferred from the reservoir $S_2 - S_1 < 0$.

10.1. Change in Entropy for Cycles

Shown in Figure IIa.10.1, are three cycles. Figure IIa.10.1(a) shows a cycle in which heat is transferred from a heat reservoir at high temperature to another heat reservoir at lower temperature. Figure IIa.10.1(b) shows the cycle for a heat engine. Finally, Figure IIa.10.1(c) shows a cycle for a heat pump. The goal is to find the change in entropy for each cycle. Starting with Figure IIa.10.1(a), we first note that in steady state operation, $Q_H = Q_L = Q$. The device can simply be a conducting metal, which transfers heat from the heat source to the heat sink. To find the change of entropy for this cycle, we use Equation IIa.10.4:

$$\Delta S_{\text{System}} + \Delta S_{\text{Surroundings}} = \Delta S_{\text{System}} + \Delta S_{\text{Heat source}} + \Delta S_{\text{Heat sink}} \geq 0$$

noting that the device operates in a cycle, hence, $(\Delta S)_{\text{System}} = 0$. Therefore, the change in entropy becomes:

$$\Delta S_{\text{Heat source}} + \Delta S_{\text{Heat sink}} = -\frac{Q}{T_H} + \frac{Q}{T_L} \geq 0 \quad \text{IIa.10.7}$$

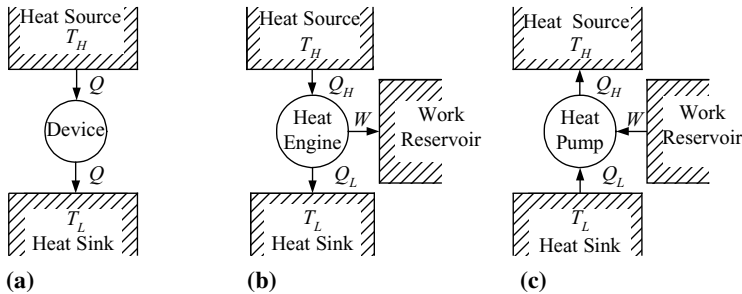


Figure IIa.10.1. Two reservoirs for (a) heat transfer, (b) heat engine, (c) heat pump

For the heat transfer to take place, the above relation must be satisfied. Since the absolute value of Q is greater than zero, it requires that $1/T_H + 1/T_L \geq 0$ or $T_H \geq T_L$. This conclusion satisfies our intuition based on experimental observations that heat flow from the hot to the cold system and if temperatures are the same then there is no heat transfer. This also supports the Clausius statement of the second law.

Example IIa.10.1. Consider two heat reservoirs, one at 550 C and another at 20 C. These reservoirs are connected by a device, resulting in a rate of heat transfer between the two reservoirs equal to 2700 MW. Find the rate of increase in the entropy of the universe as a result of this process.

Solution: To find the rate of entropy increase, we use Equation IIa.10.7:

$$\Delta S = -\frac{Q}{T_H} + \frac{Q}{T_L} = Q\left(\frac{1}{T_L} - \frac{1}{T_H}\right) = 2700\left(\frac{1}{20 + 273} - \frac{1}{550 + 273}\right) = 5.93 \text{ MW/K}$$

Let's now examine the entropy change for the heat engine. We know that for a heat engine,

$$Q_H - Q_L = W$$

Substituting for the entropy change of the heat source and heat sink and noting that for a work reservoir $\Delta S_{\text{Work reservoir}} = 0$, we obtain:

$$\Delta S_{\text{Heat source}} + \Delta S_{\text{Heat sink}} = -\frac{Q_H}{T_H} + \frac{Q_L}{T_L} + 0 \geq 0 \quad \text{IIa.10.8}$$

If the heat engine operates in a reversible process, then we can write:

$$-\frac{Q_H}{T_H} + \frac{Q_L}{T_L} = 0$$

From the above relation we conclude that $Q_L/Q_H = T_L/T_H$. If this conclusion is substituted in Equation IIa.9.1, it results in the Carnot efficiency as given by Equation IIa.9.2. It is evident that a 100% efficiency is obtained if $T_L = 0$ K. In practice T_L is about 288 K (15 C, 60 F). Therefore, it is important to increase T_H , which has its own limitations as discussed in Section 9. The conclusion that resulted in obtaining Equation IIa.9.2 also supports the Kelvin-Planck statement of the second law of thermodynamics.

The reader may try the same method used for Figures IIa.10.1(a) and IIa.10.1(b) to obtain the change of entropy for the heat pump of Figure IIa.10.1(c).

10.2. Change in Entropy for Closed Systems

We defined the closed system as a system with constant mass. Hence, in all thermodynamic processes only heat and work can cross the boundary of the system. To find the change in entropy of a closed system, we use the following inequality:

$$\Delta S_{\text{System}} + \Delta S_{\text{Surroundings}} = \Delta S_{\text{System}} + \Delta S_{\text{Heat reservoir}} + \Delta S_{\text{Work reservoir}} \geq 0 \quad \text{IIa.10.9}$$

The change in the entropy of the work reservoir is zero. The change in the entropy of the heat reservoir (*HR*) is given in Equation IIa.10.6 as $\Delta S_{\text{HR}} = Q_{\text{HR}}/T_{\text{HR}}$. Therefore, for a closed system, $\Delta S_{\text{System}} + Q_{\text{HR}}/T_{\text{HR}} \geq 0$. Whether heat is transferred from the heat reservoir to the system or from the system to the heat reservoir, we always have $Q_{\text{System}} = -Q_{\text{HR}}$, substituting we find $\Delta S_{\text{System}} - Q_{\text{System}}/T_{\text{HR}} \geq 0$. To find the differential change in entropy for a differential change in state, we replace ΔS by dS , Q_{System} by δQ_{System} , and T_{HR} by $T + dT$ of the system. If we ignore $dSdT$, then Equation IIa.10.9 simplifies to:

$$dS \geq \frac{\delta Q}{T}$$

It is apparent that the entropy increase is larger than the $\delta Q/T$ due to irreversibility. Should we add the lost work to the left-hand side, the inequality can be replaced by the equal sign. To do so, we consider two processes for the system, namely, a reversible and an irreversible process. To be able to apply the first law of thermodynamics to both processes and have the same change in the total energy of the system, we must require an identical change in the state for both processes. We start with the first law for the reversible process;

$$\delta Q_{\text{rev}} = dE + \delta W_{\text{rev}}$$

Similarly, we write the first law for the irreversible process:

$$\delta Q_{\text{irr}} = dE + \delta W_{\text{irr}}$$

Canceling dE between the two equations, we obtain:

$$\delta Q_{\text{rev}} = \delta Q_{\text{irr}} + \delta W_{\text{rev}} - \delta W_{\text{irr}} = \delta Q_{\text{irr}} + \delta I$$

where the incremental irreversibility δI is given by $\delta I = \delta W_{\text{rev}} - \delta W_{\text{irr}}$. For the reversible path we can write $dS = \delta Q_{\text{rev}}/T$. If we then substitute for $\delta Q_{\text{rev}} = TdS$, divide by T , and rearrange we obtain:

$$dS = \frac{\delta Q_{\text{irr}}}{T} + \frac{\delta I}{T} \quad \text{IIa.10.10}$$

Equation IIa.10.10 shows that the change in entropy for a closed system is solely due to the heat transfer and the irreversibility. To minimize the change in entropy, both terms should be minimized.

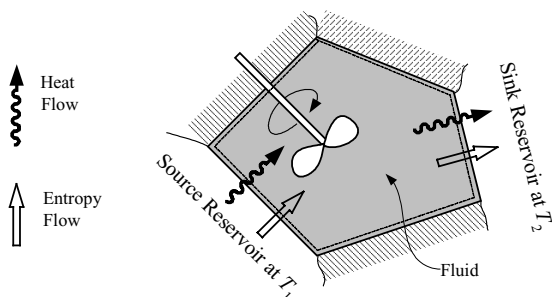


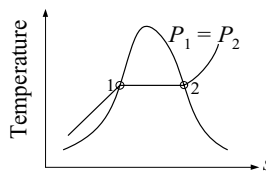
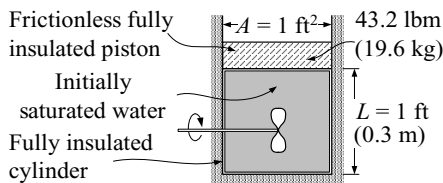
Figure IIa.10.2. Entropy transfer and production for a closed system

We are now set to examine the relation between irreversibility (I) and the measure of entropy production (σ). For this purpose, we consider the closed system of Figure IIa.10.2 in which its contents, either gas or liquid, is stirred by the action of the paddle wheel. The entropy production due to system irreversibility, such as friction, is equal to σ . Heat is introduced to the system from the hot reservoir at T_1 (transferring entropy into the system equal to Q_1/T_1) and is rejected to the cold reservoir at T_2 (transferring entropy out of the system equal to Q_2/T_2). The change in entropy can be written as:

$$S_2 - S_1 = \sum_j \frac{Q_j}{T_j} + \sigma \quad \text{IIa.10.11}$$

where j is an index to include all boundaries participating in heat transfer into or out of the system, including $j = 0$, to the surroundings. In Figure IIa.10.2, $j = 2$. The right side of Equation IIa.10.11 consists of two terms. The first term accounts for the entropy transfer into or out of the system due to exchanges with the heat reservoirs and the second term accounts for entropy production. Differentiating Equation IIa.10.11 and comparing with Equation IIa.10.10, we conclude that $\sigma = I/T$. This conclusion confirms our expectation that neither σ nor I is a property of the system as the value of both quantities depends on the type of process the system would go through.

Example IIa.10.2. A cylinder contains saturated water. The piston is frictionless and free to move. We heat the water by a mixer adiabatically to produce saturated vapor. Find the entropy produced in this process.



Solution: Boiling takes place in a closed system in a process, which is isobaric and adiabatic. Find pressure $P_{water} = P_{atm} + (Mg/A) = 14.7 + (43.2/144) = 15$ psia.

Find water mass as $m = \rho V = 59.8$ lbm (27.1 kg).

Since $Q = 0$, then Equation IIa.10.11 simply becomes $\Delta S = \sigma$. Hence, $\sigma = m(s_2 - s_1)$. Substituting, we find:

$$\sigma = m(s_g - s_f) = 59.8 \times (1.7551 - 0.3137) = 86.19 \text{ Btu (91 kJ)}.$$

10.3. Useful Work, Optimum Useful Work, and Irreversibility (Closed Systems)

If the total work obtained from a closed system is W , we define the useful work as the portion of the total work that excludes the expansion or contraction work involved with the surroundings, being at pressure P_o . In differential form, we have $\delta W_{use} = \delta W - P_o dV$. We can derive a relation for δW_{use} using the first and the second law. The first law $\delta Q = dE + \delta W$ as given by Equation IIa.6.1 can be written as:

$$\delta Q_o + \sum_j \delta Q_j = dE + \delta W_{use} + P_o dV$$

where the heat transfer term is expanded to include heat transfer to or from the surroundings and heat transfer to or from a heat source or heat sink reservoirs. Also the work transfer term is expanded to include the useful work and the expansion or contraction work with the surroundings. Writing Equation IIa.10.11 in differential form and expanding, we find:

$$dS = \frac{\delta Q_o}{T_o} + \sum_j \frac{\delta Q_j}{T_j} + \delta \sigma$$

where term $\sum_j Q_j/T_j$ is expanded to account for the surroundings separately hence, in the above equation $j \neq o$. We now find δQ_o from the second law equation (i.e., $\delta Q_o = T_o dS - \sum_j Q_j/T_j - \delta \sigma$) and substitute it into the first law equation to obtain $T_o dS - \sum_j Q_j/T_j - \delta \sigma + \sum_j \delta Q_j = dE + \delta W_{use} + P_o dV$. Solving for δW_{use} , we find:

$$\delta W_{use} = -dE - P_o dV + T_o dS + \sum_j \delta Q_j (1 - T_o/T_j) - T_o \delta \sigma$$

Useful work is optimum in the absence of any irreversibility. Thus, the *optimum useful work* is given as:

$$\delta W_{use, opt} = -dE - P_o dV + T_o dS + \sum_j \delta Q_j (1 - T_o/T_j)$$

We now integrate this equation between states 1 and 2, divide by total mass, and ignore *K.E.* and *P.E.* to get:

$$w_{use, opt} = -(u_2 - u_1) - P_o(v_2 - v_1) + T_o(s_2 - s_1) + \sum_j q_j(1 - T_o/T_j) \quad \text{IIa.10.12}$$

where $q = Q/m$. Recall that the difference between W_{use} and $W_{use, opt}$ lies in the irreversibility of the process. Therefore, the irreversibility per unit mass basis becomes:

$$w_{use, opt} - w_{use} = I/m = T_o \sigma / m \quad \text{IIa.10.13}$$

Example IIa.10.3. A rigid tank of 2 m^3 contains air at 0.4 MPa and 310 K . We now heat up the tank from a heat source at 800 K until the air temperature in the tank reaches 620 K . The surrounding atmosphere is at 1 bar and 288 K . Find a) useful work, b) optimum useful work, and c) the irreversibility of the process.

Solution: a) Since $V = \text{constant}$ and no shaft crosses the boundary, $w_{use} = 0$. Find m and P_2 for part b:

The process is isochoric; $P_2 = P_1 T_2 / T_1 = 0.8 \text{ MPa}$. Also $m = PV / RT = 0.4 \text{E}3 \times 2 / (0.287 \times 310) = 9 \text{ kg}$

b) To find $w_{use, opt}$ we need, (u_1, u_2) , (v_1, v_2) , and (s_1, s_2) . We find these properties in the following steps:

$$v_2 = v_1 = V / m = RT / P = (8.314 / 28.97) \times 310 / 0.4 \text{E}3 = 0.22 \text{ m}^3 / \text{kg}.$$

$$s_2 - s_1 = c_p \ln(T_2 / T_1) - R \ln(P_2 / P_1) = 1 \times \ln(620 / 310) - 0.287 \times \ln(0.8 / 0.4) = 0.494 \text{ kJ/kg} \cdot \text{K}$$

$$u_2 - u_1 = c_p(T_2 - T_1) = 0.72(620 - 310) = 223 \text{ kJ/kg. We find } Q \text{ from the first law: } Q / m - w_{use} = 223 \text{ kJ/kg.}$$

$$w_{use, opt} = -(223) + 288 \times 0.494 - 0.1(0) + 223(1 - 288 / 800) = 62 \text{ kJ/kg.}$$

$$\text{Thus } W_{use, opt} = 62 \times 9 = 558 \text{ kJ}$$

c) $I = W_{use, opt} - W_{use} = 558 - 0 = 558 \text{ kJ}$. This may be viewed as the work that could not be used.

Example IIa.10.4. Saturated steam condenses to saturated water in a cylinder fitted with a frictionless piston. Find a) work, b) useful work, c) optimum useful work, and d) irreversibility associated with this process.

Data: $V_1 = 2 \text{ ft}^3$ (0.057 m^3), $P_1 = 100 \text{ psia}$ (0.69 MPa), $P_o = 14.7 \text{ psia}$ (1 bar), $T_o = 525 \text{ R}$ (291 K).

Solution: We find $T_1 = 327.82 \text{ F}$, $v_g = 4.431 \text{ ft}^3 / \text{lbm}$, and $v_f = 0.0177 \text{ ft}^3 / \text{lbm}$. Thus, $m = 2 / 4.431 = 0.45 \text{ lbm}$

$$\text{a) The process is isobaric; } W = \int_1^2 P dV \quad W = Pm(v_2 - v_1) = 100 \times 144 \times 0.45(0.0177 - 4.431) / 778 = -36.7 \text{ Btu}$$

$$\text{b) } W_{use} = W - P_o m(v_{fg}) = -36.7 - 14.7 \times 144 \times 0.45(-4.4133) / 778 = -31.3 \text{ Btu}$$

$$\text{c) } w_{use, opt} = -(u_2 - u_1) - P_o(v_2 - v_1) + T_o(s_2 - s_1) + \sum_j q_j(1 - T_o / T_j).$$

In this problem, $j = o$. Substituting, we get:

$$w_{use, opt} = 807 + (14.7 \times 144 \times 4.4133 / 778) + 525(-1.1284) = 807 + 12 - 592.4 = 226.6 \text{ Btu/lbm.}$$

$$\text{d) } I = mw_{use, opt} - W_{use} = 0.45 \times (226.6) - (-31.3) = 133.3 \text{ Btu (140.6 kJ). This is the loss of work production.}$$

10.4. Change in Entropy for Control Volumes

Change in the entropy of open systems can be readily obtained from Equation IIa.10.10 noting that in open systems, entropy may be brought into the system by crossing the boundary of the system through the inlet ports. Similarly, entropy may leave the system through the outlet ports, hence, for open systems:

$$dS = \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \sum_j \frac{dQ_j}{T_j} + \sum_{C.V.} \frac{I}{T} \quad \text{IIa.10.14}$$

where we have generalized the equation for the change of entropy by considering all $\delta Q/T$ terms in the control volume to account for variation of temperature within the control volume. Similarly, we considered all the lost work due to the internal irreversibility in the control volume. Equation IIa.10.14 can be readily modified for unsteady state conditions:

$$\frac{dS_{C.V.}}{dt} = \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \sum_{C.V.} \frac{\delta \dot{Q}_{C.V.}}{T} + \sum_{C.V.} \frac{\dot{I}}{T} \quad \text{IIa.10.15}$$

It is evident from Equation IIa.10.15 that, for irreversible processes in a control volume, we would have:

$$\frac{dS_{C.V.}}{dt} \geq \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \sum_{C.V.} \frac{\delta \dot{Q}_{C.V.}}{T} \quad \text{IIa.10.16}$$

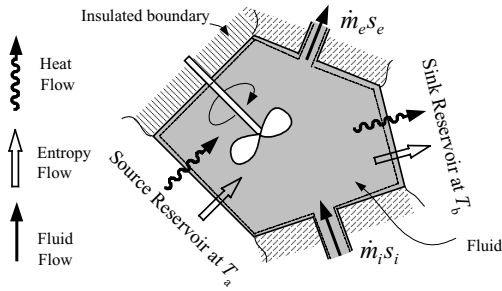
For steady flow ($\dot{m}_i = \dot{m}_e$) and steady state ($dS_{C.V.}/dt = 0$) processes we can write:

$$\dot{m}(s_e - s_i) \geq \sum_{C.V.} \frac{\delta \dot{Q}_{C.V.}}{T} \quad \text{IIa.10.17}$$

where the equals sign applies to reversible processes. For isentropic processes, $\delta \dot{Q} = 0$ hence, $s_i = s_e$.

Similar to the closed system, where we made a parallel between Equations IIa.10.10 and IIa.10.11, we are now set to find the parallel with Equation IIa.10.15 for open systems or control volumes. We do this in conjunction with Figure IIa.10.3, showing the transfer of entropy into and out of the control volume by both heat and mass transfer. We can intuitively derive the rate of change of entropy of a control volume. This is equal to the summation of the total rate of entropy transferred into the system (by heat and mass transfer), plus the rate of entropy production in the system, minus the summation of the total rate of entropy transferred out of the system. The mathematical expression of the entropy rate balance for the control volume is:

$$\frac{dS_{C.V.}}{dt} = \left(\sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e \right) + \left(\sum_j \frac{\dot{Q}_j}{T_j} \right) + \dot{\sigma}_{C.V.} \quad \text{IIa.10.18}$$



$$\begin{aligned} \text{Rate of entropy transfer by heat flow:} & \quad \frac{\dot{Q}_a}{T_a} - \frac{\dot{Q}_b}{T_b} \\ \text{Rate of entropy transfer by fluid flow:} & \quad \dot{m}_i s_i - \dot{m}_e s_e \\ \text{Rate of entropy production by friction:} & \quad \dot{\sigma} \end{aligned}$$

Figure IIa.10.3. Entropy transfer and production for an open system

Comparing Equation IIa.10.18 with Equation IIa.10.15 indicates that $\dot{\sigma}_{C.V.} = \dot{I}_{C.V.} / T$. At steady state, $dS_{C.V.}/dt = 0$, hence, Equation IIa.10.18 becomes:

$$\left(\sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e \right) + \sum_j \left(\dot{Q}_j / T_j \right) + \dot{\sigma}_{C.V.} = 0 \quad \text{IIa.10.19}$$

10.5. Useful Work, Optimum Useful Work, and Irreversibility (Control Volumes)

Recall that for the closed systems, we combined the first and second law, Equations IIa.6.1 and Equation IIa.10.11 to obtain Equation IIa.10.12. Similarly, we may combine Equation IIa.6.3 and Equation IIa.10.18 to obtain the equation for useful work for a control volume. In specific, for flow entering and leaving a rigid control volume under steady state conditions (i.e. Equations IIa.6.5 and IIa.10.19) we find that the optimum useful shaft work at steady state (ss) operation is given by (see Problem 117):

$$\dot{W}_{opt,ss} = \sum_j \dot{m}_i \left(h_i + V_i^2 / 2 + gz_i - T_o s_i \right) - \sum_j \dot{m}_e \left(h_e + V_e^2 / 2 + gz_e - T_o s_e \right) + \sum_j \dot{Q} \left(1 - \frac{T_o}{T_j} \right) \quad \text{IIa.10.20}$$

If flow through the control volume at steady state condition is also steady flow, then the irreversibility per unit mass flow rate is given by Equation IIa.10.16 and Equation IIa.10.20 becomes:

$$w_{opt,ss} = \sum_j \left(h_i + V_i^2 / 2 + gz_i - T_o s_i \right) - \sum_j \left(h_e + V_e^2 / 2 + gz_e - T_o s_e \right) + \sum_j q \left(1 - \frac{T_o}{T_j} \right) \quad \text{IIa.10.21}$$

Example Ila.10.5. A globe valve is used to throttle steam in a steady state process from 10 MPa and 360 C to 4 MPa. The valve is fully insulated. Find the rate of entropy production in this process.

Solution: The throttling process in the valve is iso-enthalpic, $h_e = h_i$.

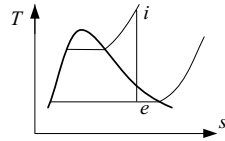
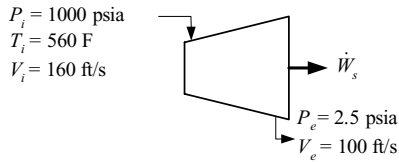
At $P_i = 10$ MPa and $T_i = 360$ C, $s_i = 6.006$ kJ/kg·K and $h_i = 2962.1$ kJ/kg.

At $P_e = 4$ MPa and $h_e = 2962.1$ kJ/kg, $s_e = 6.362$ kJ/kg·K.

From Equation Ila.10.16 with $Q = 0$ and $\dot{m}_i = \dot{m}_e$:

$$\dot{\sigma} / \dot{m} = 6.362 - 6.006 = 0.356 \text{ kJ/kg·K}$$

Example Ila.10.6. Superheated steam enters a turbine at $P_1 = 1000$ psia, $T_1 = 560$ F, and $V_1 = 160$ ft/s. Steam expands isentropically to a pressure of 2.5 psia and leaves the turbine at 100 ft/s. The inlet flow area of the turbine is $A_i = 22$ ft². Find a) work delivered by the turbine, b) entropy produced in this expansion process, c) the optimum useful work, and d) irreversibility. For the surroundings use $P_o = 14.7$ psia and $T_o = 70$ F.



Solution: a) We first find superheated properties at 1000 psia and 560 F from the steam tables:

P_i (psia)	T_i (F)	v_i (ft ³ /lbm)	h_i (Btu/lbm)	s_i (Btu/lbm·R)
1000	560	0.4668	1210.4	1.4082

We then find the exit conditions at 2.5 psia from the steam tables as follows:

P_e (psia)	$h_{f,e}$ (Btu/lbm)	$h_{g,e}$ (Btu/lbm)	$s_{f,e}$ (Btu/lbm·R)	$s_{g,e}$ (Btu/lbm·R)
2.5	101.71	1119.4	0.188	1.9029

To find the work performed by the turbine we use the first law, Equation Ila.6.5. Since there is no heat loss from the turbine, the change in elevation is negligible and the work is delivered under steady flow, steady state condition, this equation simplifies to:

$$\dot{m} \left(h_i + \frac{V_i^2}{2} \right) = \dot{W}_s + \dot{m} \left(h_e + \frac{V_e^2}{2} \right)$$

To find the rate of work delivered, we need to find h_e and \dot{m} . The exit enthalpy is given by $h_e = h_{f,e} + x_e h_{fg,e}$. Having $h_{f,e}$ and $h_{fg,e}$, we must find x_e . This is obtained from the isentropic expansion of steam in the turbine:

$$s_i = s_e = s_{f,e} + x_e s_{fg,e}$$

$$x_e = \frac{s_i - s_{f,e}}{s_{g,o} - s_{f,e}} = \frac{1.4082 - 0.188}{1.9029 - 0.188} = \frac{1.2202}{1.7149} = 0.712$$

$h_e = h_{f,e} + x_e(h_{g,e} - h_{f,e}) = 101.71 + 0.712(1119.4 - 101.71) = 826.3$ Btu/lbm. We find the mass flow rate from $\dot{m} = \rho_i V_i A_i = (1/0.4668) \times 160 \times 22 = 7540.7$ lbm/s. Thus, the power produced by the turbine is obtained as:

$$\dot{W}_s = \dot{m} \left((h_i - h_e) + \left(\frac{V_i^2}{2} - \frac{V_e^2}{2} \right) \right) =$$

$$7540.7 \times \left((1210.4 - 826.3) + \left(\frac{160^2 - 100^2}{2 \times 32.2 \times 778} \right) \right) = 2.898\text{E}6 \text{ Btu/s}$$

b) In an isentropic process, no entropy is produced. This is confirmed by Equation Ila.10.16, since $s_1 = s_2$ and $\dot{Q} = 0$, therefore, $\sigma_{C.V.} = 0$.

c) and d) In this problem, $w_{use} = w_{use, opt}$ and $I = 0$.

11. Exergy or Availability

Our goal is to determine the maximum work that can be obtained in a work-producing process from a given system. Such system may contain various forms of energy including kinetic, potential, chemical, electrical, and nuclear. The necessary and sufficient conditions for obtaining the maximum work from a system are satisfied in a reversible process that brings the system to the *dead state*. The dead state for pressure (P) and temperature (T) of the system is when P and T reach P_o and T_o of the surroundings, respectively. When this occurs, the system is in chemical, mechanical, and thermal equilibrium with the surroundings. Such equilibrium with the surroundings is required if work can be extracted by any means. For example, if system contains kinetic energy then its velocity should be brought to zero. Similar argument applies to potential energy, etc. Next we investigate the availability (exergy) of closed systems and of control volumes focusing on the systems that contain only mechanical and thermal energies.

11.1 Availability (Exergy), Closed Systems

Shown in Figure Ila.11.1 is a closed system containing a hot gas with the frictionless piston held in place by a stop. We now remove the stop and let the gas expand. The work delivered by the piston in an infinitesimal move is $\delta W_{use} = \delta W - P_o dV$. To bring temperature down to that of the surrounding (T_o), δQ heat must

be rejected to the surrounding. If used in a reversible machine, this amount of heat can produce work given by:

$$\delta W_{\text{equivalent}} = -\delta Q(1 - T_o/T)$$

where T is the system temperature during the process, ranging from $T_1 \leq T \leq T_o$. Therefore the net work in this expansion process is found by deducting the work corresponding to the heat rejection from the useful work (i.e. $\delta W_{\text{net}} = \delta W_{\text{use}} - \delta W_{\text{equivalent}}$). Substituting, we obtain $\delta W_{\text{net}} = \delta W - P_o dV - \delta Q(1 - T_o/T)$. Since we assumed a reversible expansion due to the frictionless piston, this is the maximum work that can be obtained in this process. For closed systems, we consider the system internal energy, substitute for δW from the first law ($\delta W = \delta Q - dU$), and for δQ from the second law ($\delta Q = TdS$) to find the relation for the infinitesimal work as $\delta W_{\text{use,opt}} = -dU - P_o dV + T_o dS$. Integrating, the net work in this process is found as:

$$W_{\text{use,opt}} = (U - U_o) + P_o(V - V_o) - T_o(S - S_o) \quad \text{IIa.11.1}$$

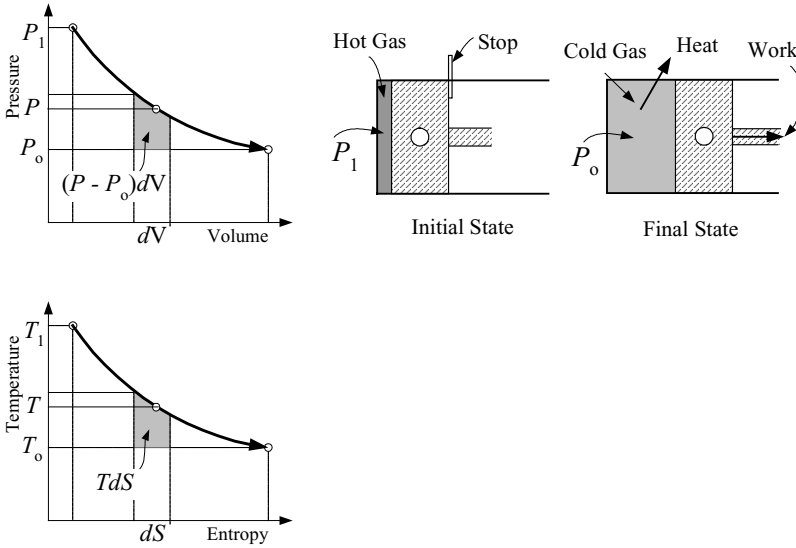


Figure IIa.11.1. A process for bringing a closed system from state 1 to the dead state

where the *opt* in the subscript is added to emphasize the reversible process. The work obtained in Equation IIa.11.1 is the closed system availability (Φ). Written on a specific basis, the specific availability becomes:

$$\phi = (u + P_o v - T_o s) - (u_o + P_o v_o - T_o s_o) \quad \text{IIa.11.2}$$

Equation IIa.11.2 can alternatively be written as:

$$\phi = (u - u_o) + P_o(v - v_o) - T_o(s - s_o) \quad \text{IIa.11.3}$$

so that $\Phi = m\phi$. Assuming the surroundings condition is at atmospheric pressure (14.7 psia = 101 kPa) and room temperature (77 F = 25 C), then the specific exergy, ϕ can be considered as yet another property.

Example IIa.11.1. A cylinder contains 5 kg of air (treated as an ideal gas) at 1 MPa and 350 C. The piston is held in place by a stop pin. Find the maximum useful work when the frictionless piston is set free to move.

Solution: Find $v = RT/P = (8.314/28.97) \times (350 + 273)/1E3 = 0.179 \text{ m}^3/\text{kg}$ and $v_o = 0.847 \text{ m}^3/\text{kg}$

$$P_o(v - v_o) = 101 \times (0.179 - 0.847) = -67.468 \text{ kJ/kg}$$

$$u - u_o = c_v(T - T_o) = 0.7165 \times (350 - 25) = 232.86 \text{ kJ/kg}$$

$$s - s_o = c_v \ln(T_1/T_o) + R \ln(v_1/v_o) = 0.7165 \times \ln(623/298) + (8.314/28.97) \times \ln(0.179/0.847) = 0.0823 \text{ kJ/kg}$$

$$\phi = (u - u_o) + P_o(v - v_o) - T_o(s - s_o) = 232.86 - 67.468 - 298 \times 0.0823 = 140.86 \text{ kJ/kg}$$

$$\Phi = 5 \times 140.86 = 704.33 \text{ kJ. } W_{use,opt} = 704.33 \text{ kJ.}$$

Change in Availability

We can readily derive the change in availability for closed systems by combining the first and the second law of thermodynamics, Equations IIa.6.1 and IIa.10.11, respectively. The first law, $E_2 - E_1 = \int_1^2 \delta Q - W$ added to the second law, while multiplied by T_o , and rearranged results in:

$$\Phi_2 - \Phi_1 = \int_1^2 \left(1 - \frac{T_o}{T_b} \right) \delta Q - [W - P_o(V_2 - V_1)] - T_o \sigma \quad \text{IIa.11.4}$$

Equation IIa.11.4 demonstrates that the change in availability is due to the availability transfer (the first three terms in the right side) and the availability destruction (the fourth term in the right-hand side). The terms representing availability transfer itself consists of availability transfer associated with heat (the first term in the right side) and the availability transfer associated with work (the second and third term in the right side).

Example IIa.11.2. A piston-cylinder assembly contains m lbm of saturated water at 212 F. We now add heat to the cylinder from a reservoir at temperature T , in a reversible process (the frictionless piston is free to move) until all water becomes saturated steam. Verify Equation IIa.10.18 for this process.

Solution: On the one hand, change in specific availability is given by:

$$\Delta\phi = (u_g - u_f) + P_o(v_g - v_f) - T_o(s_g - s_f).$$

On the other hand, $\Delta\Phi$ from Equation IIa.11.4 for $\sigma = 0$ is:

$$\Delta\Phi = (1 - T_o/T)Q - [W - P_o(V_2 - V_1)].$$

The availability transfer due to work is:

$$W - P_o(V_2 - V_1).$$

Since expansion work is given by $W = P\Delta V$, if we substitute for W , we find:

$$P\Delta V - P_o(\Delta V) = 0.$$

Therefore, $\Delta\Phi = (1 - T_o/T)Q$. Heat transfer is given by $Q/m = h_{fg}$. Thus, $\Delta\Phi = (1 - T_o/T)mh_{fg}$. This can be written as

$$\Delta\Phi/m = \Delta\phi = h_{fg} - T_o(h_{fg}/T) = (u_{fg} + P_o v_{fg}) - T_o s_{fg}.$$

Since availability can be viewed as a property of the system, which by definition, is independent of the path and depends only on the end states, we can find the change in system availability when the system goes from state 1 to state 2 as $\phi_2 - \phi_1 = (u_2 - u_1) + P_o(v_2 - v_1) - T_o(s_2 - s_1)$. Upon comparing with Equation IIa.10.12, we find the change in availability given as:

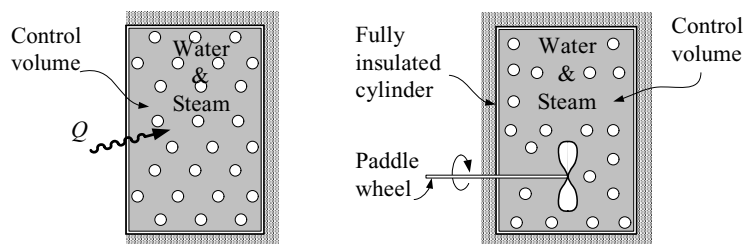
$$\phi_1 + \sum_j \dot{q}_j (1 - T_o/T) = W_{use,opt} + \phi_2$$

this equation can also be written as:

$$w_{use,opt} = -(\phi_2 - \phi_1) + \sum_j q_j (1 - T_o/T_j) \quad \text{IIa.11.5}$$

If the change in availability has been solely due to work transfer, Equation IIa.11.5 for adiabatic processes simplifies to $w_{use,opt} = -(\phi_2 - \phi_1)$.

Example IIa.11.3. A tank of 2 ft³ contains two-phase mixture of water and steam at 200 psia with $x = 10\%$. We want to increase the mixture temperature to 456.3 F by one of the following two processes. Find the irreversibility if a) heating the tank from a reservoir at 600 F and b) insulating the tank and using a paddle wheel. The surrounding pressure and temperature are 15 psia and 59 F.



Solution: We first find the mixture properties at the initial and the final states:

P (psia)	T (F)	v_f (ft ³ /lbm)	v_g (ft ³ /lbm)
200	381.8	0.01839	2.2873
450	456.3	0.01954	1.0318

u_f (Btu/lbm)	u_g (Btu/lbm)	s_f (Btu/lbm·R)	s_g (Btu/lbm·R)
354.8	1113.7	0.5438	1.5454
435.7	1118.9	0.636	1.4738

$v_2 = v_1 = 0.01839 + 0.1(2.2689) = 0.245 \text{ ft}^3/\text{lbm}$. Thus $x_2 = (0.245 - 0.0195)/1.01224 = 22.3\%$.

$m = V/v = 2/0.245 = 8.154 \text{ lbm}$. We now find change in availability for a closed system:

$$\phi_2 - \phi_1 = (u_2 - u_1) + P_o(v_2 - v_1) - T_o(s_2 - s_1) = (157.37) + 0 - 519 \times 0.1789 = 64.52 \text{ Btu/lbm or } \Delta\Phi = 526 \text{ Btu}$$

a) Since $W = 0$, we find Q from the first law, $Q = \Delta u = 8.154(157.37) = 1283.2 \text{ Btu}$. Next we find $W_{\text{use, opt}}$:

$$(W_{\text{use, opt}})_a = -\Delta\Phi + Q(1 - T_o/T) = -526 + 1283.2(1 - 519/1060) = 129 \text{ Btu}.$$

b) Since $Q = 0$, we find W from the first law, $W = -\Delta u = 8.154(157.37) = -1283.2 \text{ Btu}$. We find $W_{\text{use, opt}}$:

$$(W_{\text{use, opt}})_a = -\Delta\Phi = -526 \text{ Btu}.$$

In both process, $I = W_{\text{use, opt}} - W$. Thus $I_a = 129 - 0 = 129 \text{ Btu}$ and $I_b = -526 - (-1283.2) = 757.2 \text{ Btu}$

Since $I_b \gg I_a$, from a thermodynamic view point, heat transfer is preferred than using work to produce heat.

We may also use Equation IIa.10.19 to define a second law effectiveness (ζ) for a work producing process:

$$\zeta = \frac{w}{w_{\text{use, opt}}} = \frac{w}{-(\phi_2 - \phi_1) + \sum_j q_j (T_o / T_j)} \quad \text{IIa.11.6}$$

Example IIa.11.4. A cylinder contains steam at 3 MPa and 320 C. The frictionless piston is set free to move. After expansion, steam pressure and temperature drop to 0.7 MPa and 180 C. The work resulting from this expansion is 185 kJ/kg and the sink reservoir to exchange heat is at 100 C. Find the effectiveness.

Solution: We set up the following table for the data

P (MPa)	T (C)	v (m^3/kg)	u (kJ/kg)	s (kJ/kg·K)
3.00	350	0.0850	2788.4	6.6245
0.75	180	0.2847	2599.8	6.7880

Use the first law to find q_{1-2} gives:

$$q_{1-2} = w_{1-2} + (u_2 - u_1) = 185 + (2599.8 - 2788.4) = -3.6 \text{ kJ/kg}$$

$$w_{1-2} = \phi_1 - \phi_2 + q_{1-2}(1 - T_o/T_s) = (u_1 - u_2) + P_o(v_1 - v_2) - T_o(s_1 - s_2) + q_{1-2}(1 - T_o/T_s)$$

$$w_{1-2} = (2788.4 - 2599.8) + 101(0.085 - 0.2847) - 298(6.788 - 6.6245) - 3.6(1 - 298/373) = 256.77 \text{ kJ/kg}$$

$$\zeta = 185/256.77 = 72\%.$$

11.2. Availability (Exergy), Control Volumes

We define the flow exergy for open systems in a manner similar to that of the closed systems except for the fact that the specific flow exergy must account for the potential and kinetic energies of the fluid, knowing that at the dead state the system should reach the velocity of the surroundings (zero) and the same elevation as the surroundings. As a result, the exergy for a control volume per unit mass basis is defined as:

$$\psi = (h - h_o) - T_o(s - s_o) + (V^2 / 2) + g(Z - Z_o) \quad \text{IIa.11.7}$$

where Z_o is the elevation at the dead state. Therefore, the change in the inlet and exit availabilities becomes:

$$\Delta\psi = (h_e - h_i) - T_o(s_e - s_i) + (V_e^2 - V_i^2) / 2 + g(Z_e - Z_i) \quad \text{IIa.11.8}$$

In most practical applications, the kinetic and potential energies are neglected compared to the fluid enthalpy. Using Equation IIa.11.8, the optimum useful work at steady state for a control volume can be obtained if we stipulate multiple input and exit ports and an exchange of heat and work with the surroundings and heat reservoirs. This work is the difference between the availabilities of the inlet and exit streams plus the work associated with the exchange of heat with heat reservoirs:

$$\dot{W}_{opt,ss} = \sum_i \dot{m}_i \psi_i - \sum_e \dot{m}_e \psi_e + \sum_j \dot{Q}_j \left(1 - \frac{T_o}{T_j} \right) \quad \text{IIa.11.9}$$

It then follows that the irreversibility associated with the steady flow of fluids through a control volume with multiple ports, while exchanging heat and work with the surroundings and heat reservoirs, is given as:

$$\dot{I}_{c.v.} = \sum_i \dot{m}_i \psi_i - \sum_o \dot{m}_o \psi_o + \sum_j \dot{Q}_j \left(1 - \frac{T_o}{T_j} \right) - \dot{W}_{c.v.} \quad \text{IIa.11.10}$$

Example IIa.11.5. Steam enters a fully insulated turbine at 800 psia and 550 F. Steam leaves the turbine at 10 psia with $x_e = 80\%$. Find the following items: a) work delivered by the turbine, b) the maximum useful work, c) the availability of the exit stream, d) the effectiveness, and e) the irreversibility. Use $T_o = 530$ R.

Solution: a) From the first law with $q = 0$, we find $w = h_i - h_e = 1230.1 - 964.94 = 283.2$ Btu/lbm.

b) $w_{opt,ss} = \psi_i - \psi_e = (h_i - h_e) - T_o(s_i - s_e)$. Thus $w_{opt,ss} = 283.2 - 530(1.447 - 1.487) = 304.42$ Btu/lbm

c) $\psi_e = (h_e - h_o) - T_o(s_e - s_o)$. For h_o and s_o of the dead state we use saturated properties for a subcooled liquid corresponding to T_o :

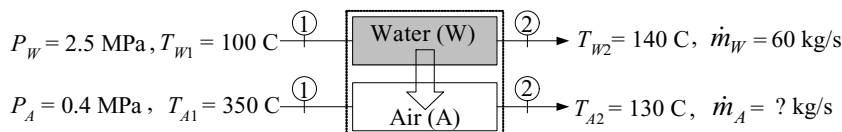
$\psi = (964.94 - 38.05) - 530(1.487 - 0.0745) = 178.3$ Btu/lbm.

d) $\zeta = w/w_{opt,ss} = 283.2/304.42 = 93\%$ and finally

e) $I_{c.v.} = w_{opt,ss} - w = 304.42 - 283.2 = 21.22$ Btu/lbm

In the thermal design of turbines it is important to minimize the availability of the exit stream to increase the effectiveness. In the above example, the irreversibility associated with the adiabatic expansion of steam is due to the increase in entropy during the expansion of the steam in various stages of the turbine.

Example Ila.11.6. Water is heated by a stream of hot air in a heat exchanger as shown. Use the data as given in the figure to find the system irreversibility. Ignore pressure drop in both streams. Use $T_o = 295$ K.



Solution: We ignore the *K.E.* and *P.E.* of both streams and treat air as an ideal gas. Stream availabilities are:

$$\Delta\psi_W = (h_{W2} - h_{W1}) - T_o(s_{W2} - s_{W1}) = (590.52 - 420.85) - 295(1.7369 - 1.3050) = 42.26 \text{ kJ/kg}$$

$$\Delta\psi_A = (h_{A2} - h_{A1}) - T_o(s_{A2} - s_{A1}) = (130 - 350) - 295(-0.3365) = -120.73 \text{ kJ/kg}$$

$\Delta\dot{\psi}_W = \dot{m}_W \Delta\psi_W = 60 \times 42.26 = 2535.6 \text{ kJ}$. We find \dot{m}_A from an energy balance for the heat exchanger:

$$\dot{m}_W(h_{W2} - h_{W1}) = \dot{m}_A c_{p,A}(T_{A1} - T_{A2}). \text{ Thus } \dot{m}_A = 60(590.52 - 420.85)/(350 - 130) = 46.27 \text{ kg/s}$$

$$\Delta\dot{\psi}_A = \dot{m}_A \Delta\psi_A = 46.27 \times (-120.73) = -5586.6 \text{ KJ}$$

$$\dot{I}_{c.v.} = \sum_i \dot{m}_i \psi_i - \sum_o \dot{m}_o \psi_o - \dot{W}_{c.v.} = \dot{m}_W \psi_W + \dot{m}_A \psi_A = 2535.6 - 5586.6 = -3051 \text{ kJ}$$

Example Ila.11.7. Cooling water at a rate of 170,000 lbm/s enters the condenser of an electric utility from a lake at 60 F and leaves at 75 F. The plant also produces exhaust gases at a rate of 450 lbm/s and 455 F. Find the more wasteful stream leaving this electric plant. Use $T_o = (60 + 460) = 520$ R.

Solution: We ignore the *K.E.* and *P.E.* and take the exhaust gases to be air, behaving as an ideal gas. Thus, for both water as compressed liquid and air as ideal gas, $\Delta h \approx c_p \Delta T$. We need to compare ψ_W with ψ_A .

Since the process for both streams is isobaric, $s - s_o = c_p \ln(T/T_o)$.

$$\psi_W = \dot{m}_W [c_{pW}(T_W - T_o) - c_{pW}T_o \ln(T_W/T_o)] = 1.7E5[1.0(75 - 60) - 1.0 \times 520 \ln(535/520)] = 36,086 \text{ Btu}$$

$$\psi_A = \dot{m}_A [c_{pA}(T_A - T_o) - c_{pA}T_o \ln(T_A/T_o)] = 450[0.24(455 - 60) - 0.24 \times 520 \ln(915/520)] = 10,924 \text{ Btu}$$

The cooling water carries more untapped energy than the stack gases by a factor of 3.

QUESTIONS

Section 1

- What are the primary dimensions?
- Mention three derived units.
- What is barometric pressure? What is the absolute pressure of total vacuum?
- Pressure of a gas container is 2 psig. What is the absolute pressure of the gas container?
- What is a pure substance? Is water a pure substance?
- What is the difference between a system and its surrounding?

Section 2

- Explain the difference between ideal, perfect, and real gases.
- Comparing the Van der Waals equation with the ideal gas law, can we conclude that the former accounts for the existence of gas molecule, hence, reduces the available volume in a gas container ($v - c_2$)?
- Comparing the Van der Waals equation with the ideal gas law, which equation of state accounts for the intermolecular attractive force ($P + c_1/v^2$)?
- By accounting for the net attraction of the molecules within a gas on an individual molecule, does the Van der Waals equation account for the reduction in the impulse the molecule would have otherwise exerted on the wall of a gas container?

Section 3

- What is degree of subcooling?
- Consider the saturation temperature of water, $T_{sat} = f(P)$. Does T_{sat} increase, remain the same, or decrease with increasing pressure?
- What is the difference between a polytropic and an isentropic process?
- A system has pressure P_1 at one instant and pressure P_2 at another instant. Is the change in pressure an exact differential?
- Is it fair to say that any change in the properties of a system in any process is always an exact differential?

Section 4

- Give an example for the “insulated system”
- What is the difference between control mass and control volume?
- Define control surface
- Since no mass crosses the boundaries of a closed system, how can its energy content change?
- Which of heat, work, and total energy of a system is an exact differential?
- Is there any work associated with the rotation of a shaft in a well lubricated journal bearing?

Section 5

- What is the difference between steady flow and steady state?

- Consider heating up a steel rod. Is this a steady state process? Give an example for a steady state process.
- Consider a compressor as a control volume. The air density changes as air flows through the inlet towards the outlet. If $dm_{C.V.}/dt = 0$ and $dE_{C.V.}/dt = 0$, is this process a steady state process?

Section 6

- What is the difference between a nozzle and a diffuser?
- What is the difference between a turbine and a compressor?
- What is the difference between a compressor and a pump?

Section 7

- Is it a good idea to insulate compressors and turbines?
- Is it fair to say that water density remains constant from the suction to the discharge of a pump?
- Does one control volume allow determination of the temperature distribution inside the control volume?

Section 8

- What is the key assumption in the dynamic analysis of mixing tanks?
- How do we find the mass flow rate through a control valve while discharging gas filled rigid vessels?
- How can we add heat to a rigid vessel in an isobaric process?

Section 9

- Can any process that does not violate the first law of thermodynamics be reversed?
- Which process takes place more readily, conversion of work to heat or conversion of heat to work?
- What is the Kelvin-Planck statement on the transfer of energy from a heat source to a heat sink?
- What is the difference between internal and external irreversibility?
- We bring a hot block of metal in contact with a cold block of metal. Is the heat transfer between these two blocks of metal reversible?
- Is any reversible process necessarily an adiabatic process?
- What is the difference between a reversible and an isentropic process?
- Is it possible to transfer heat from a heat sink to a heat source? Doesn't this violate the second law?
- A heat engine is operating between T_H and T_C . Which temperature do you change to increase efficiency?
- What is the function of a heat pump? How do you define the coefficient of performance for a heat pump?

Section 10

- Is entropy, like energy, conserved in any process?

- Describe unavailability in the context of dissipative effects of an irreversible process.
- What is the change in entropy of a work reservoir ($dS_{\text{Work reservoir}} = ?$)
- What is the proportionality constant for the change of entropy of a heat reservoir ($dS_{\text{Heat reservoir}} = ?$)
- Support the Clausius statement of the second law using entropy change for a device that works in a cycle and transfers heat from the heat source to the heat sink.
- Does entropy change in an isolated system?
- In what ways does entropy change for a closed system? Answer the same question for an open system.
- How do you define useful work, the optimum useful work, and irreversibility?

PROBLEMS

Sections IIa.1 and IIa.2

1. A system is left alone for a long time. During this time, no mass, no heat, and no work have crossed its boundary. Is this system at equilibrium?
2. A system is left alone for a long time. During this time no mass, no heat, and no work have crossed its boundary. Are properties of this system (i.e., such macroscopically measurable quantities as pressure, volume, and temperature) independent of time?
3. Find the weight in lbf of a substance having a mass equal to 4536 g. [Ans.: 10 lbf].
4. a) At certain flow conditions, the maximum mass flow rate of an ideal gas through a cross section, known as the critical flow, is given by $\dot{m} = bP/\sqrt{T}$. Find units of b if units of \dot{m} , P and T are lbm/s, psia, and degree Rankine, respectively.
b) The critical flow of saturated steam per unit area may be estimated from a relation known as Rateau correlation: $G = P[16.367 - 0.96\log_{10}P]/1000$. In this correlation, units of P and G are psia and lbm/s-in², respectively. Convert this relation so that for P in MPa, we obtain G in kg/s-cm².
5. Partial vacuum is often measured in torr where 1 torr is 133.322 Pa or 1.316E-3 atm. A vacuum pump is used to bring pressure in a tank down to 2.8 torr. Find the tank pressure in cm Hg and cm H₂O.
6. Find the *K.E.* of a substance having a mass of 2 kg and moving at a velocity of 5 m/s. [Ans. 50 J].
7. Find the *K.E.* of a substance having a mass of 2 lbm and moving at a velocity of 5 ft/s. [Ans.: 1.55 ft·lbf = 2E-3 Btu].

8. Find the kinetic and potential energies of a ball having a mass of 2 lbm and travelling at 5 ft/s at an elevation of 10 ft above the ground. [Ans.: 0.776 ft·lbf and 20 ft·lbf].
9. Find the *K.E.* of a 5000 lbm car traveling at 55 miles per hour. [Ans.: 0.5E6 ft·lbf]
10. A 100 lb rock is lifted to a height of 100 ft. Find the change in the potential energy of the rock in Btu.
11. Find the atmospheric pressure in feet of water and cm of mercury. The specific weight of mercury is 13.6. [Ans.: 33.92 ft and 76 cm-Hg].
12. Water at atmospheric pressure in a standpipe is supplied to a hydrant. Water pressure at the hydrant must be 65 psig. Find the height of the standpipe with respect to the hydrant to meet this requirement. Use $\rho_{\text{water}} = 62.4 \text{ lbm/ft}^3$. [Ans.: 150 ft]
13. A U-tube containing mercury is used as a manometer. This manometer is now connected to a container containing gas at 0.404 atm. Find the difference in the mercury height in the U-tube after being connected to the container. [Ans.: 1 ft].
14. A gas is drawn in a pipe by a vacuum pump. The manometer reads -3 in Hg. Find the gas gage pressure in inches of mercury and the absolute pressure in psia.
15. A mercury manometer reads a pressure of 5 in Hg. We now want to substitute a manometer filled with oil having a density of 45 lbm/ft³. Find the reading on the oil-filled manometer. [Ans.: 94.3 in Oil].
16. A liquid of unknown density is used in a manometer. When $P_{\text{atm}} = 14.7 \text{ psia}$, we read $H_1 = 6.72 \text{ m}$. Find the liquid density in lbm/ft³. [Ans.: 96 lbm/ft³].

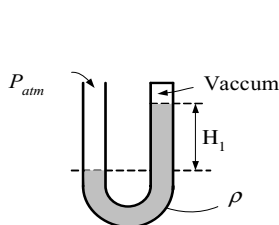


Figure for Problem 16

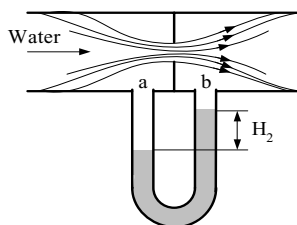


Figure for Problem 17

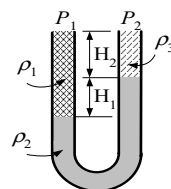
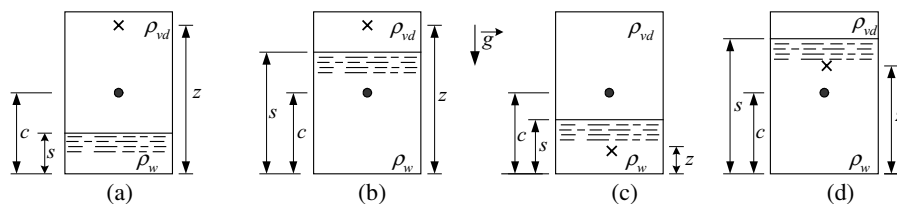


Figure for Problem 18

17. The liquid in Problem 16 is now used in measuring the pressure drop of water flowing through a thin plate orifice. For $H_2 = 60 \text{ cm}$, find pressure drop over the orifice. [Ans.: 0.46 psi].
18. For the heights and densities in the U-tube, find $P_1 - P_2$ in terms of H_1 , H_2 , ρ_1 , ρ_2 , and ρ_3 .
19. A tank contains a pool of water (density ρ_w) and a mixture of water vapor and water droplets (density ρ_{vd}). Pressure at height c (center of the tank) is given. Find

pressure at height z of each tank in terms of P_c and given heights and densities.

[Ans.: a) $P_z = P_c + (c - z) \rho_{vd}g$].



20. Find the Kmols of ammonia (NH_3) that is equivalent to 34 kg of NH_3 . [Ans.: 2 kmol]

21. Find the lb-moles of CO_2 contained in 120 g of CO_2 . [Ans.: 6.02E-3 lb-mole].

22. Find the mass of air in a 1 m^3 tank. Pressure in the tank is 1 MPa and air temperature in the tank is 40 C. [Ans. 11.11 kg].

23. A pressure vessel having a volume of 171 ft^3 contains 1.523 lbmoles of helium at a pressure of 7 atm. Find the temperature of helium in this tank. [Ans.: 620 F].

24. In this problem we want to compare the prediction of three equation of states for gases. These are the ideal gas, $Pv = RT$, the Van Der Waals $(P + c_1/v_2)(v - c_2) = RT$, and the Beattie-Bridgeman equation:

$$P = \frac{RT}{\bar{v}^2} \left(1 - \frac{c}{\bar{v}T^3} \right) (\bar{v} + B) - \frac{A}{\bar{v}^2}$$

For this comparison, use CO_2 at $T = 300 \text{ K}$ and $v = 0.0040 \text{ m}^3/\text{kg}$. Compare the results with the value of 6.6 MPa obtained experimentally. Note that in the Beattie-Bridgeman equation v is in m^3/kmol , T is in K, and P is in kPa. Also $A = A_0(1 - a/v)$ and $B = B_0(1 - b/v)$. For CO_2 , $A_0 = 507.2836$, $a = 0.07132$, $B_0 = 0.10476$, $b = 0.07235$, and $c = 660,000$. [Ans. $P_{IG} = 14.17 \text{ MPa}$, $P_{VDW} = 6.95 \text{ MPa}$, and $P_{BB} = 6.741 \text{ MPa}$].

25. Use a Maxwell relation and show that the change in entropy of an ideal gas is given as:

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Section IIa.3

26. Plot water density as a function of temperature in the range of 32 F to 100 F. Find the peak water density.

27. Find the enthalpy of a water mixture at 2000 psia and a quality of 50%. [Ans.: 905 Btu/lbm].

28. Use the steam tables and find the specific volume of water at a) $P = 550$ psia and $T = 580$ F, b) $P = 600$ psia and $T = 180$ F, c) $P = 500$ psia and $u = 800$ Btu/lbm, d) $P = 500$ psia and $h = 1000$ Btu/lbm.

29. Use the steam tables and find steam quality for a) $T = 120$ C and $v = 0.6$ m³/kg, b) $P = 2250$ psia, $h = 1000$ Btu/lbm, and c) $P = 10$ MPa, $v = 0.015$ m³/kg.

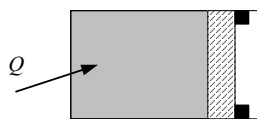
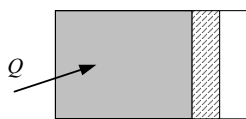
30. Use the steam tables and find the specific entropy of water at a) $P = 10$ MPa and $T = 180$ C, b) $P = 2$ MPa and $T = 370$ C, c) $P = 5$ MPa and $u = 1200$ kJ/kg, d) $P = 5$ psia and $h = 1200$ F.

31. Use the steam tables and find the temperature and the thermodynamic state of water at $P = 7.5$ MPa and $h = 1200$ kJ/kg.

32. For water, we are given $P = 350$ psia and $T = 134.604$ F. Can we find other thermodynamic properties such as v , u , h , and s ? Explain your answer.

Sections Ila.4 through Ila.8

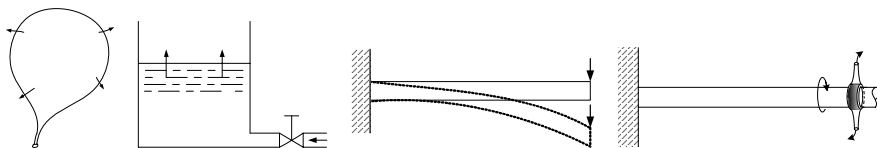
33. Heat is added to a cylinder as shown in the figure. Find the type of process in both cases.



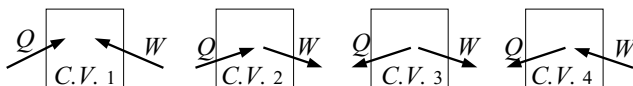
34. Write the conservation of mass and the first law of thermodynamic for a closed system undergoing a cycle.

35. Determine if any work is associated with the following actions and the type of the work if applicable:

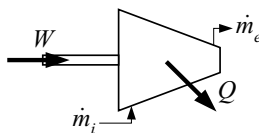
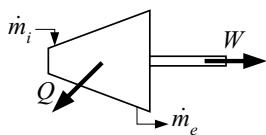
a) inflating a balloon, b) filling a tank from the bottom, c) depressing the free end of a cantilever and d) twisting a free end of a cantilever.



36. Select the sign of the heat and work terms in the equation for the first law of thermodynamics written for a closed system (i.e., $E_1 + Q = W + E_2$) given the following cases:



37. Select the sign of the heat and work terms in the equation for the first law of thermodynamics written for a control volume given the following cases:



38. Specify the type of work in the following examples, a) crushing an empty soda can, b) pulling a spoon out of a honey jar, c) cranking an engine, d) pumping water, e) turning a shaft inside a lubricated bearing.

39. The solar collector shown in Figure I.7.5 is used to provide domestic hot water. Assuming a person needs about 20 gallon/day (76 liter) of hot water at 140 F (60 C), find the collector surface area to meet this demand. Use a tap water temperature of 60 F (15.5 C). For solar radiation, use a heat flux (radiant energy divided by the collector surface area) of 236 W/m^2 . Due to the collector thermal properties, only 80% of the sun's energy is available to warm the flowing water in the solar collector. [Ans.: 1.57 m^2].

40. We want to evaluate the operation of the relief valve in the radiator cap of a car on a hot summer day while the car is driven up a hill. Before the engine is started, water is at atmospheric pressure and room temperature (P_1 and T_1 , respectively). At this condition, the volume of water in the engine block, radiator, water pump, and the connecting hoses is V_1 . The volume between the water surface and the top of the radiator is ΔV . We now start the engine and begin driving the car on the long road leading to the hill. The relief valve opens when the pressure reaches P_H . 1) Plot the expansion and the pressurization processes on the T - v diagram of Figure IIa.3.1(c) and 2) Explain how you find the amount of heat transferred to the water when pressure reaches P_H . For this evaluation you may assume: a) water is incompressible (i.e., changes in water density are negligible), and b) air is removed so that water expansion is an isobaric process.

41. A tank contains air treated as an ideal gas initially at 100 psia and 200 F. We now heat up this tank until its pressure reaches 110 psia. Find the air temperature at this pressure.

42. A cylinder equipped with a piston contains saturated steam at 2 MPa. We now compress the steam in an isentropic process until its volume becomes equal to $2/3$ of its original volume. Find the steam pressure, temperature, and its thermodynamic state.

43. A cylinder contains air at 150 psia and 250 F. The air is kept in the cylinder with a tightly fit piston. At this state, the cylinder volume is 5 ft^3 . We now compress the air, treated as an ideal gas, while heat is removed so that compression takes place in constant pressure until the air volume becomes 2 ft^3 . Find the amount of heat removed from the cylinder.

44. In this problem we want to find the work associated with the compression of an ideal gas. A cylinder-piston assembly contains 2 kg of air, treated as an ideal gas. The air in the cylinder is initially at 10 bar pressure and 25 C. We now push the piston and compress the air but keep the pressure at 10 bar by letting heat

transfer out of the cylinder. Find the work delivered to the system when volume reaches 1/3 of the initial volume.

45. In this problem we want to find the work associated with the torsion of a solid bar. If τ is the applied torsion resulting in an elemental twist of $d\theta$, the work delivered to the bar is $\delta\dot{W} = \tau d\dot{\theta}$. Consider the shaft of an electric motor receiving a torque equal to 35 N m at a constant angular velocity of 1200 rpm. Find the rate of work delivered by the electric motor to the shaft. [Ans.: $\dot{W} \cong 4.4$ kW].

46. In this problem we want to find electric work. A current of I amp at a voltage of V volt, is associated with a power of VI . Find the work associated with charging a battery for 5 hours at a voltage of 12 V and a current of 2.5 A. [Ans.: 540 kJ].

47. In this problem, we want to find the work associated with a change in the surface area of fluids. As described in Chapter III, surface tension as force per unit length, is a liquid property tending to maintain liquid surface. The work associated with a differential change in the liquid surface area is found as $\delta W = 2\sigma dA$, where σ is surface tension. Find the work required to blow a bubble 5 cm in diameter from soapy water. At 25 C temperature, soapy water has a surface tension of about $\sigma = 0.073$ N/m. [Ans.: 1.15E-3 J].

48. In this problem, we want to find the heat produced in a gearbox. The work brought into the system at steady state condition by the high-speed drive shaft is 1 MW. The work carried away on the low-speed shaft is 0.95 MW. Find the amount of heat produced. [Ans. 50 kW].

49. To compress air in a cylinder, 1000 Btu of energy is required. This compression process results in the internal energy of the air to increase by 100 Btu. Find the amount of heat transfer involved in the process. Is this amount of heat transferred to the cylinder or transferred from the cylinder? [Ans.: -900].

50. The steam in a cylinder undergoes a process in which 1000 kJ of heat is transferred to a cylinder. The addition of heat to the cylinder results in the internal energy of the steam to be increased to 800 kJ. Find the amount of work delivered to the piston. [Ans.: 200 kJ].

51. Find the thermal power of the PWR of the nuclear ship *Savannah*. The reactor operated at 1,750 psia. The coolant entered the reactor vessel at rate of 9.4E6 lbm/h and a temperature of 497 F and exited at 519 F. [Ans. 71.33 MWth].

52. Pressurized air at a rate of 4.5 kg/s flows in a rectangular duct. The air pressure and temperature at a point in the duct is measured as 33 C and 250 kPa. The duct cross section is a rectangle of 50 cm by 20 cm. Find a) the volumetric flow rate, b) the mass flux, c) the average velocity at this location. [Ans.: c) 15.8 m/s].

53. Liquid sodium enters the core of a liquid metal fast breeder reactor (LMFBR) at 400 C and leaves at 560 C. The reactor operates at 750 MW. Find the sodium flow rate. $c_p = 0.3$ Btu/lbm·F. [Ans.: 12,471 lbm/s].

54. Use the Maxwell relations to show that, for an isentropic process of an ideal gas, $Pv^\gamma = \text{constant}$ where γ is given as $\gamma = c_p/c_v$. [Hint: Use $ds = (c_v dT/T) + R (dv/v) = 0$ and $ds = (c_p dT/T) - R (dP/P) = 0$. Cancel dT/T and integrate.]

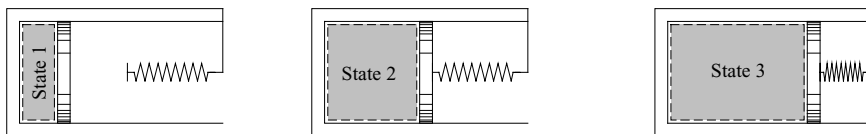
55. Use the equation of state for ideal gases in conjunction with the Maxwell relations to derive an alternative equation to Equation IIa.3.6 (calculation of the change in the entropy of a system in a reversible process. [Ans.:

$$s_2 - s_1 = \int_{T_1}^{T_2} c_p (dT/T) - R \ln(P_2/P_1)] .$$

56. A tank at atmospheric pressure contains two inlets and one outlet port. The first inlet port has a flow area of 0.05 ft^2 and the flow area of the second inlet port is 0.025 ft^2 . Water enters the first inlet port at 5 ft/s and 100 F . Water enters the second inlet port at 8 ft/s and 175 F . Water leaves the tank at a rate of 2 lbm/s . Find the rate of change of the tank water level.

57. We compress air at 1 MPa and 150 C to a pressure of 5 MPa in an isentropic process. Treat air as an ideal gas and find its temperature at this pressure. [Ans.: $T_2 \approx 240 \text{ C}$].

58. A mixture of water and steam is contained in a cylinder equipped with a well-fitted leak-tight piston of cross sectional area A . At state 1, the mixture is at pressure P_1 having a steam quality of x_1 and a volume of V_1 . Heat is added to the cylinder until the piston just touches the spring. At this stage, the volume of the cylinder content is $V_2 = V_1 + \Delta V$. We keep adding heat to the cylinder. The piston would travel further to the right and start compressing the spring. We terminate the heating process when the pressure of the cylinder content reaches P_3 . Write a procedure from which P_2 , T_2 , and T_3 can be determined. Assume a linear spring ($F = k_s x$) with known k_s .

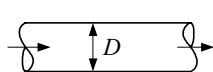


59. Show that for steady-flow, steady state isentropic process of an ideal gas the work from going from state 1 to state 2 is found from:

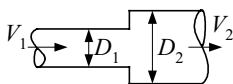
$$W_{12} = \frac{\gamma R T}{\gamma - 1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \right]$$

60. Flow enters a heat exchanger at a rate of $25\text{E}6 \text{ lbm/h}$ and a density of 1.5 slugs/ft^3 . There are 9000 tubes in this heat exchanger. If flow is distributed evenly among the tubes, find the flow velocity in each tube. Use a diameter of 0.63 in for all the tubes. [Ans.: 7.38 ft/s].

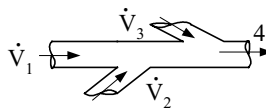
61. Water at 20 C enters the pipe of Figure (a) at a velocity of 2.5 m/s . The pipe has an inside diameter of 5 cm . Find the mass flow rate, mass flux, and volumetric flow rate. [Ans. 4.9 kg/s , $2497.5 \text{ kg/m}^2\text{-s}$, $4.9\text{E-}3 \text{ m}^3/\text{s}$].



(a)



(b)



(c)

62. Show that for the pipe of Figure (b) flow velocity at the exit is given by $V_2 = V_1 (D_2/D_1)^2$.

63. For the pipe in Figure (c), find mass flow rate (\dot{m}_4) and velocity (V_4) at the exit of the pipe.

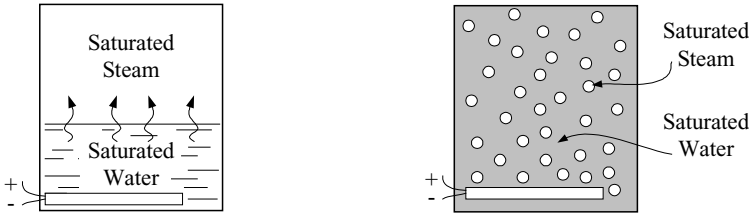
64. The mass flow rate through the core of a 2×4 PWR (i.e., 2 hot legs and four cold legs) is $62.82\text{E}6$ kg/h. The inside diameter (I.D.) of the hot leg is 1 m. Estimate the I.D. of the cold leg. Data: $T_{HL} = 320$ C, $T_{CL} = 288$ C, and $P = 15.5$ MPa. [Hint: $V_{HL} = V_{CL}$].

65. The core of a nuclear reactor produces 2772 MWth at 2155 psia. The volumetric flow rate through the core is given as $122.71\text{E}6$ GPM. The core outlet temperature is 604 F. Find the core inlet temperature. [Hint. Guess T_{in} , find T_{avg} , find $\rho_{avg}(P, T_{avg})$, find \dot{m} , update T_{avg} and continue iteration]. [Ans.: 549 F].

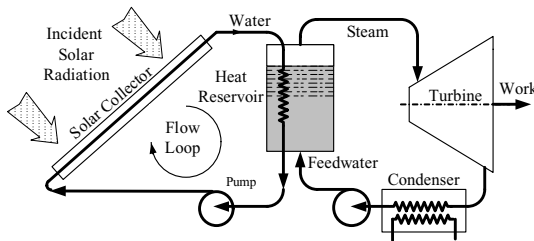
66. Consider the pressurizer of a PWR, having an internal volume of 1500 ft³. The pressurizer contains 750 ft³ of water and steam at 2000 psia at equilibrium. Due to a turbine trip, an in-surge of 100 GPM and 600 F enters the pressurizer for 5 minutes. Find the temperature of the water region after termination of the in-surge. Assume perfect mixing between the in-surge and water in the water region. Ignore work due to boundary change and heat transfer with the steam region. [Ans. $T_2 = 662.5$ F].

67. The pressurizer of a PWR is at 2250 psia. Water through the letdown line leaves the pressurizer at a rate of 44 GPM and enters the volume control tank (VCT) for 30 minutes. If no other process has taken place in either tank, use the data below and find the change in water level in the two tanks. Assume instantaneous and perfect mixing in VCT. Data: $V_{\text{Pressurizer}} = 1500$ ft³, $(V_{\text{water}})_{\text{Pressurizer}} = 750$ ft³, $A_{\text{Pressurizer}} = 50$ ft², $V_{\text{VCT}} = 1000$ ft³, $(V_{\text{water}})_{\text{VCT}} = 385$ ft³, $A_{\text{VCT}} = 44.3$ ft², $T_{\text{VCT}} = 150$ F.

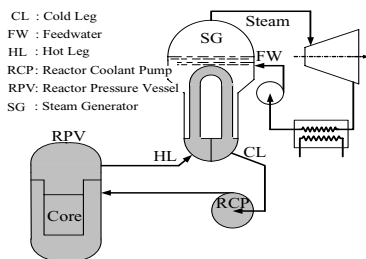
68. A rigid vessel is filled with saturated water and steam. In the left figure, water and steam constitute two separate regions while in the right figure the water and steam make a homogenous mixture. Under what condition can the right figure represent the left figure?



69. We plan to design a 1 MWth solar power station, as shown in the figure. Solar radiation heats up the circulating water in the solar collectors, which is then transferred to a heat reservoir to produce steam. The reservoir is maintained at 1 MPa. Dry, saturated steam, after expansion in the turbine, is cooled down in the condenser and is pumped back to the reservoir. Find the steady state mass flow rate of feedwater and of steam. The steam enthalpy at the exit of the turbine is 2300 kJ/kg. List the advantages and drawbacks of this design. Changes in the kinetic and potential energies are negligible. [Ans.: 2 kg/s].



70. Total heat in the primary side of a PWR, as shown in the figure, is 2700 MWth. This amount of heat consists of the fission heat produced in the core and the reactor coolant pump (RCP) heat. The steam generator is maintained at a pressure of 900 psia. The enthalpy of feedwater entering the steam generator is 430 Btu/lbm. Find the steady state mass flow of the dry saturated steam leaving the steam generator towards the turbine. [Ans.: 11.7E6 lbm/s].



71. Shown in Figure (a) is the flow path of a BWR vessel. Find the steam mass flow rate (\dot{m}_g) in terms of total reactor core power (\dot{Q}), feedwater enthalpy (h_d), and thermodynamic properties at vessel pressure.

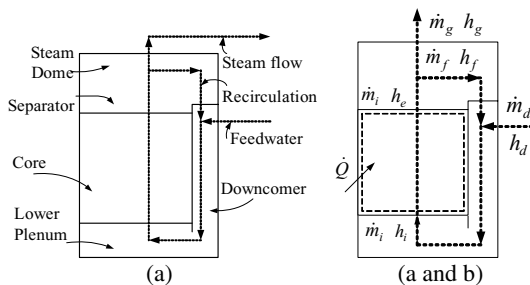


Figure for Problem 71

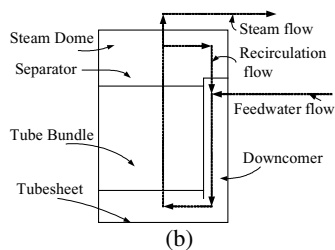


Figure for Problem 73

[Ans: $\dot{m}_g = \dot{Q} / (h_g - h_d)$].

72. In Problem 71, defining the recirculation flow rate as $r = \dot{m}_f / \dot{m}_g$, show that $r = (1 - x_e) / x_e$ where x_e is the steam quality at the core exit.

73. Shown in Figure (b), is the flow path in the secondary side of a PWR steam generator. For the steady state operation, find the steam mass flow rate in terms of the following parameters:

h_{in} : water enthalpy at the inlet to the tube bundle region

h_f : saturated water enthalpy at the steam generator pressure

h_{fg} : latent heat of vaporization at the steam generator pressure

\dot{Q}_{Core} : total rate of heat transfer in the core,

\dot{m}_i : mass flow rate in the tube bundle region,

N_{SG} : number of steam generators (N_{SG})

[Ans.: $\dot{m}_s = \{(\dot{Q}_{Core} / N_{SG}) - \dot{m}_i(h_f - h_{in})\} / h_{fg}$].

74. A PWR steam generator at steady state operation produces dry, saturated steam. Pressure in the tube bundle region is 6 MPa (875 psia). The PWR power plant is equipped with two steam generators and is producing a total electric power of 810 MWe at a thermal efficiency of 30%. Use a feedwater enthalpy of 1007 kJ/kg (433 Btu/lbm), a recirculation ratio of 3.3 to find:

a) the feedwater flow rate entering the steam generator downcomer, b) the steam flow rate c) the flow rate entering the tube bundle region, d) the recirculation flow rate entering the downcomer region, e) water enthalpy entering the tube bundle region, f) mixture enthalpy at the exit of the tube bundle region, g) the degree of subcooling at the inlet to the tube bundle region.

75. Start with Equation IIa.6.3 and show that, for rigid control volumes with no internal heat generation, the first law for control volumes simplifies to:

$$\sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2g_c} + \frac{g}{g_c} Z_i \right) + \sum \dot{Q} = \sum \dot{W}_s + \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2g_c} + \frac{g}{g_c} Z_e \right) + \frac{d}{dt} \left[m \left(u + \frac{V^2}{2g_c} + \frac{g}{g_c} Z \right) \right]$$

IIa.6.3

76. Saturated steam enters a turbine at 7 MPa and a rate of 6E3 kg/h. Steam leaves turbine at 7 bar and $x_e = 85\%$. There is a total of 20 kW heat loss from the turbine. Find the power developed by this turbine. [Ans.: 0.93 MW].

77. Hot water enters the steam generator tubes of a PWR at a rate of 138.5E6 lbm/h, pressure of 2250 psia and temperature of 600 F. Water leaves the tubes at 550 F. Steam is produced in the secondary side at 1000 psia. Find the steam mass flow rate. [Ans.: 7E6 lbm/h].

78. Find the rate of steam produced in a BWR operating at 1,600 MWth. Water enters the core from the lower plenum at a rate of 50E6 lbm/h and a temperature of 526 F. Reactor pressure is 1050 psia. [Ans.: 6E6 lb/h].

79. A high temperature gas-cooled reactor (HTGR) is designed to operate at 330 MWe with a $\eta_{th} = 39.23\%$. Helium enters the reactor at a pressure of 710 psia and temperature 760 F and leaves at 1,430 F. Find the He flow rate through the core. $c_p = 1.24$ Btu/lbm·F. [Ans.: 3.455E6 lbm/h]

80. For a PWR steam generator, we define the recirculation ratio as $R = \dot{m}_r / \dot{m}_s$, where \dot{m}_s is the steam mass flow rate and \dot{m}_r is the recirculation mass flow rate. Express R in terms of core exit average quality ($X_e = \dot{m}_s / \dot{m}$). [Ans.: $R = X_e / (1 - X_e)$].

81. Obtain an analytical solution in closed form for the set of mass and energy equations in Example IIa.8.2. [Hint, since the inlet mass flow rates and enthalpies as well as the outlet mass flow rate are all uniform with time, the rate of change of mass in the tank is constant, hence, water level is a linear function of time].

[Ans.: Mass of water in the tank as a function of time is found from

$$m(t) = \left(\sum_{i=1}^2 \dot{m}_i - \dot{m}_e \right) t + m(t=0) \quad \text{and water temperature in the tank from}$$

$$\theta = e^{-\int p(t) dt} \left\{ \left[\int q(t) e^{\int p(t) dt} dt \right] + C \right\} \quad \text{where } p(t) \text{ and } q(t) \text{ are obtained from}$$

$$p(t) = \left[\left(\sum_{i=1}^2 \dot{m}_i - \dot{m}_e \right) c_v + \left(\sum_{i=1}^2 \dot{m}_i \right) c_p \right] / m(t), \quad q(t) = \left(\sum_{i=1}^2 \dot{m}_i h_i \right) / m(t), \text{ and } C \text{ is}$$

found from the initial condition for water temperature.]

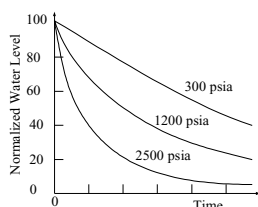
82. Find an analytical solution for Example IIa.8.2 if heat is also added to the tank at a constant rate of \dot{Q} Btu/s. [Ans.: The only modification is in $q(t)$ which be-

$$\text{comes } q(t) = \left(\dot{Q} + \sum_{i=1}^2 \dot{m}_i h_i \right) / m(t).]$$

83. If in Example IIa.8.2 heat is added to the water at a rate of 2000 Btu/s, find the time it takes for water to reach saturation at atmospheric pressure.

84. A tank contains 5 ft³ of water at 100 F and 1 atm. Heat, at a constant rate of 1,000 Btu/s, is added to the tank in an isobaric process. Find the time it takes for the last drop of water to evaporate. Properties of subcooled water at 1 atm and 100 F are $v = 0.01613$ ft³/lbm and $h = 68.04$ Btu/lbm. [Ans.: 4.6 min.]

85. A tank contains 5 ft³ of water at 100 F. Heat, at a constant rate of 1,000 Btu/s is added to the tank in an isobaric process. Find the time it takes for the last drop of water to evaporate. Solve this problem for three cases. In case 1, the tank pressure is 300 psia. In case 2, the tank pressure is 1200 psia. In case 3, the tank pressure is 2500 psia. a) What conclusion do you reach from this study? b) Assume a tank cross sectional area of 1 ft² and plot water level as a function of time for all three cases. [Ans.: a) As shown in Figures Ila.3.1(c) and Ila.3.4, latent heat of vaporization decreases as pressure increases. Hence, the tank loses water faster at higher pressures. b) the plot should have the trend shown below:



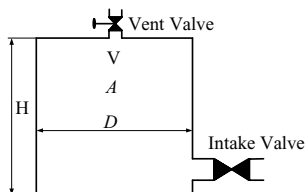
].

86. A tank of 8000 ft³ (225 m³) contains a two-phase mixture of water and steam at $P_i = 1000$ psia (7 MPa) and $x_i = 28.233\%$. Heat is now added to the tank, treated as a single control volume at a rate of:

$$\dot{Q}(t) = \dot{Q}_o e^{\alpha t}$$

where $\dot{Q}_o = 34.4$ MW and $\alpha = 18$ h⁻¹. Find the time it takes for pressure to reach 3000 psia (21 MPa). [Hint: At state 2, we have P_2 and $v_2 = v_1$]. [Ans.: 5 min].

87. A cylindrical tank has a base diameter of $D = 3$ ft and a height of $H = 7$ ft. The tank contains air at 100 psia and 100 F. The intake valve is now opened to allow 100 F water at a constant rate of 10 lbm/s to enter the tank. Assume that no air is dissolved in the water and no heat transfer takes place at the air - water and air - wall interfaces. Find air pressure in the tank 62 seconds after the intake valve is opened. Treat air as an ideal gas and the compression of air as an isentropic process. The vent valve remains closed. [Ans.: 137.2 psia].



88. A rigid vessel having a volume of V contains air, initially at a pressure of P_1 and a temperature of T_1 . An intake valve is now opened to allow pressurized air at a temperature of T_i to enter the vessel. The intake valve is closed when pressure in the vessel reaches P_2 . Use the conservation equations of mass and energy as well as the equation of state to derive a relation for the final temperature. Consider the process adiabatic and neglect any storage of heat in the tank wall. [Ans.: $T_2 = P_2 c_p T_i / \{c_v (P_2 - P_1) + (P_1 c_p T_i / T_1)\}$].

89. A rigid tank has a volume of $V = 1.0 \text{ ft}^3$ and contains air at $P_1 = 14.7 \text{ psia}$ and $T_1 = 70 \text{ F}$. An admission valve is now opened to allow pressurized air at $P_i = 100 \text{ psia}$ and $T_i = 70 \text{ F}$ enter the tank. The valve is closed when $P_2 = 30 \text{ psia}$. Find T_2 . [Ans.: $T_2 = 161 \text{ F}$].

90. The volume of the water in the secondary side of a PWR steam generator is 130 m^3 . The power deposited to the water ten minutes after the reactor is shut-down is 58 MW . If there is no feedwater delivered to the steam generator, find the time to boil the steam generator dry. The secondary side pressure is 9 MPa .

91. A tank with a volume of $V = 2 \text{ m}^3$ contains air at 3 MPa and 200 C . The vent valve is now opened. Find the tank pressure and temperature when $1/3$ of the air escapes through the vent valve. The intake valve remains closed.

92. A tank with a volume of 10 m^3 contains air at 0.1 MPa and 15 C . The intake valve is opened to allow pressurized air enter the tank at an average mass flow rate of 0.5 kg/s and temperature of 100 C . If the tank is fully insulated, find the tank pressure after 60 seconds. The vent valve remains closed. [Ans.: $P = 0.55 \text{ MPa}$, $T = 182 \text{ C}$].

93. A tank with a volume of $V = 10 \text{ m}^3$ contains air at 0.1 MPa and 15 C . The intake valve is opened to allow pressurized air enter the tank at an average mass flow rate of 0.5 kg/s and temperature of 100 C . During the charging process, heat is transferred to the atmosphere at a rate of $0.01(T - T_s) \text{ Btu/s}$ where T is the air temperature in the tank and $T_s = 10 \text{ C}$ is temperature of the surroundings. Find the tank pressure after 60 seconds. The vent valve remains closed. [Ans.: $P = 0.53 \text{ MPa}$, $T = 168 \text{ C}$].

94. A pressurized rigid vessel having a volume of 10 ft^3 is filled with air to 600 psia and 185 F . We want to vent this tank so that the final pressure drops to atmospheric pressure (14.7 psia). However, we would like to maintain the air temperature in the tank at 185 F throughout the venting process. Find the amount of heat necessary to accomplish this task. Treat air as an ideal gas with constant specific heat and ignore changes in the air kinetic and potential energies compared to its internal energy. [Ans.: 898 Btu].

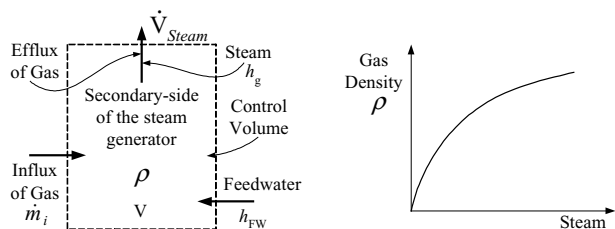
95. A rigid vessel is depressurized from 3 MPa to 101 kPa through a small vent. Heat is added in this process to maintain temperature at its initial value. The tank has a volume of 5 m^3 and its inventory is an ideal gas. Find the amount of heat required for accomplishing this task. [Ans.: $14,495 \text{ kJ}$].

96. A cylindrical pressurized vessel is filled with air to 500 psia at an initial temperature of 250 F. The vessel has a diameter of $D = 4$ ft and total volume of $V = 200$ ft³. A small, 1 inch vent valve is now opened. Find pressure (P) and temperature (T) of the gas in the vessel 20 seconds into the venting event. Treat air as an ideal gas. The tank is fully insulated. The valve discharge coefficient is 0.65. The intake valve remains closed [Hint: Flow rate through the valve should be multiplied by the specified discharge coefficient]. [Ans.: $P = 133$ psia, $T = 10$ F].

97. A cylindrical pressurized vessel is filled with air to 500 psia at an initial temperature of 250 F. The vessel has a diameter of $D = 4$ ft and total volume of $V = 200$ ft³. A small, 1 inch vent valve is now opened. Find pressure (P) and temperature (T) of the gas in the vessel 20 seconds into the venting event. Treat air as an ideal gas. The rate of heat transfer from the tank to the surroundings is estimated at $0.008(T - T_s)$ Btu/s where the surrounding is at a temperature of $T_s = 35$ F. The valve discharge coefficient is 0.65. The intake valve remains closed. [Ans.: $P = 132$ psia, $T = 6$ F].

98. The pressurizer of a PWR is a cylindrical tank having a volume of 1500 ft³. Initially, the tank is full of a mixture of water and steam. Consider this saturated mixture to be distributed uniformly in the tank at $P_1 = 2250$ psia. The initial steam quality is 70%. We now start heating the tank but would like to keep pressure at its initial value of 2250 psia. To achieve this goal, we must simultaneously remove mass from this tank. If only steam is removed by a valve at the top of the tank and the kinetic and potential energies are negligible, find the amount of the mass removed and heat added when the last drop of water boils and becomes steam.

99. A small amount of leakage exists in the steam generator of a PWR operating at the rated power of \dot{W}_{100} . A noble gas escapes the primary side and enters the secondary side at a fixed rate of \dot{m}_i . Find the density of the gas in the secondary side of the steam generator versus time. The volume of the secondary side is V .



[Hint: Since the leak is small, we treat gas as a component. Find $\rho_{100}(t)$, density of the gas at 100% power from $V(d\rho_{100}/dt) = \dot{m}_i - \rho_{100} \dot{V}_{\text{Steam}}$ where \dot{V}_{Steam} is the steam volumetric flow rate at 100% with $\rho_{100}(0) = 0$].

[Ans.: $\rho_{100}(t) = (1 - e^{-\beta t})\alpha$ where $\alpha = \dot{m}_i / \dot{V}_{\text{Steam}}$ and $\beta = \dot{V}_{\text{Steam}} / V$].

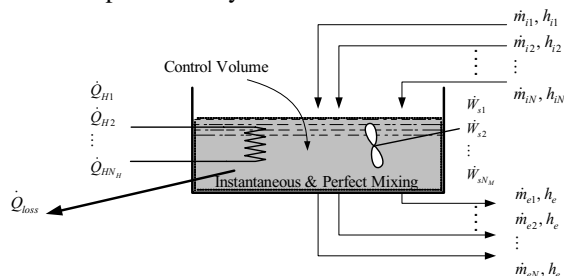
100. A small amount of leak exists in the steam generator of a PWR operating at the rated power of \dot{W}_{100} . A noble gas escapes the primary side and enters the sec-

secondary side at a fixed rate of \dot{m}_i . At θ seconds into the operation at nominal power, we reduce power to 20% of nominal. a) If the primary side and the secondary side pressure remain about the same value as at nominal power, find the steam flow rate (\dot{m}_{20}) in terms of \dot{W}_{100} , \dot{W}_{20} , and steam mass flow rate at full power (\dot{m}_{100}). b) Find the partial density of the noble gas in the secondary side (ρ_{20}), at the reduced power of 20%, versus time.

[Ans.: $\rho_{20}(t') = \alpha' + (\rho_{100}(t = \theta) - e^{-\beta' t'}) \alpha'$ where $t' = t - \theta$, $\alpha' = \dot{m}_i / \dot{V}'_{\text{Steam}}$, and $\beta' = \dot{V}'_{\text{Steam}} / V$].

[Note, the above answers assume that the noble gas is stable. If radioactive gases such as Xe-135 are involved, the Xe buildup in the primary side and decay in the secondary side must be factored in.]

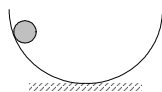
101. Consider a mixing tank that contains m_0 kg of water initially at T_0 C. This tank is fed by N feed lines carrying water at various temperatures. The mass flow rate and enthalpy of water in the feed lines are known functions of time. There are N_M mixers and N_Q heaters. The tank is poorly insulated. The rate of heat loss is given as $\beta(T_{C.V.} - T_f)$ where β has units of W/C and T_f is temperature of the surroundings. Both β and T_f are known functions of time. There are N inlet ports and M outlet ports. Assume instantaneous and perfect mixing so that water in the tank can be represented by one control volume.



a) Use other simplifying assumptions, as used in Section 8, and obtain the governing differential equations for the control volume mass and enthalpy. b) Assume a constant specific heat and obtain an analytical solution for the water temperature leaving the tank in terms of the specified forcing functions. c) Use the definitions in Chapter VIIe and obtain the solution in explicit, semi-implicit, and fully-implicit numerical schemes.

Section IIa.9 through IIa.11

102. Is the motion of the sphere a reversible process in the absence of any air resistance and friction?



103. A smooth pipe equipped with an isolation valve connects two tanks containing air. When the valve is closed $P_A > P_B$. We now open the valve until both tanks reach equilibrium. Is this a reversible process?



104. A pendulum, placed in an enclosure, is operating in a vacuum. The connecting rod is attached to a frictionless joint. Is the motion of this pendulum a reversible process?

105. An adiabatic and reversible process is an isentropic process. Can an irreversible process in which heat is allowed to transfer have no change in entropy? Clarify your answer. [Ans.: Yes].

106. Consider two heat engines operating between the same heat source and heat sink in the Carnot cycle. One heat engine uses gas and the other uses water as working fluid. Which heat engine would have higher thermal efficiency?

107. We want to heat up the contents of a closed system. We may use a paddle wheel or a heat reservoir. Thermodynamically, which method is preferred?

108. A cylinder, fitted with a frictionless piston, contains saturated steam at a specified pressure. We now let heat transfer take place from the cylinder to the surroundings until saturated steam becomes saturated water. Does this constitute a reversible process? Does this constitute an isentropic process?

109. Heat is added to a cylinder containing air. The cylinder has a volume of 0.12 m^3 and initially is at $P_1 = 1 \text{ MPa}$ and $T_1 = 50 \text{ C}$. Find a) the air pressure when the air temperature reaches 150 C , b) the amount of heat added to the cylinder, and c) the change in the air entropy. [Ans.: 1.31 MPa , 94 kJ , 0.252 kJ/C].

110. A cylinder contains 3 kg of air at $P_1 = 1 \text{ bar}$ and $T_1 = 27 \text{ C}$. In a polytropic compression, the pressure and temperature of the air are raised to $P_2 = 15 \text{ bar}$ and $T_2 = 227 \text{ C}$. Find the polytropic exponent, the final volume, the amount of compression work delivered to the system, and the amount of heat rejected to the surroundings. [Ans.: 1.23 , 0.287 m^3 , -763 kJ , and -316 kJ].

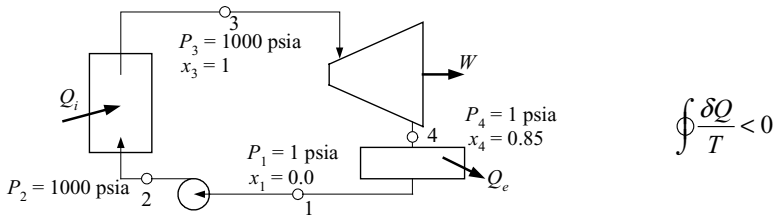
111. Thermal efficiency of a power plant is calculated as 30% . The electrical output of the plant is 1000 MWe . How much heat is transferred in the heat source to the working fluid (i.e., MWth)?

112. In a 1000 MW power plant, steam at 1000 F enters the turbine. Pressure in the primary side of the condenser is 4 psia . Find the least possible amount of heat rejected to the surroundings.

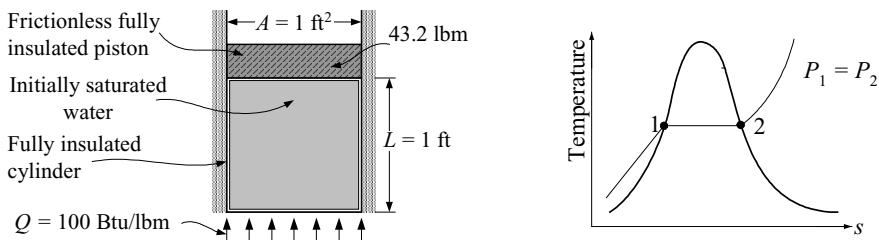
113. A power plant operates at a thermal efficiency of 32% . The rate of heat transfer in the heat source is 2700 MW . Find the power produced by the turbine.

114. A refrigerator operates at 1 kW power to maintain the temperature of the freezer compartment at -7°C while the room temperature is 25°C . The power transferred to the room from the refrigerator is 4 kW. Find the COP ($\eta_{\text{Refrigerator}}$) and compare it with the maximum COP ($\eta_{\text{Refrigerator, Carnot}}$).

115. We want to verify the validity of the data reported for the following power plant. Water leaving the condenser is saturated at $P_1 = 1$ psia. Water entering the boiler is subcooled at $P_2 = 1000$ psia and $T_2 = 79.26^\circ\text{F}$ ($h_2 = 50$ Btu/lbm). Steam leaving the boiler and entering the turbine is saturated at $P_3 = 1000$ psia. Finally, the mixture leaving the turbine and entering the condenser is at $P_4 = 1$ psia and $x_4 = 0.85$. [Hint: You must verify the Clausius inequality noting that heat is added in the boiler and rejected at the condenser].



116. Heat is added from the bottom to an otherwise well-insulated cylinder until all the initially saturated water becomes saturated steam then the heat addition is terminated and the bottom is rapidly insulated. Use the data shown in the figure to find a) the amount of work produced, b) the entropy transfer to the cylinder, and c) the entropy produced in the cylinder.



117. Derive Equation IIa.10.20 by expanding the rate of heat transfer term in Equation IIa.10.19 as:

$$\sum_j (\dot{Q}_j / T_j) = \dot{Q}_o / T_o + \sum_k (\dot{Q}_k / T_k)$$

where j is the summation over all thermal boundaries except for o (i.e., $k \neq o$). Substitute this relation into Equation IIa.10.19, solve for \dot{Q}_o and substitute into Equation IIa.6.5, where in Equation IIa.6.5 you should also make a similar expansion for the heat transfer term, $\sum_j \dot{Q}_j = \dot{Q}_o + \sum_k \dot{Q}_k$. Equation IIa.10.20 gives

the optimum useful work. Find the equation from which the useful work w_{use} can be calculated.

118. Superheated steam is throttled at steady state conditions from 10 MPa and 480 C to 6 MPa. Find the entropy production in this process.

[Ans.: 0.158 kJ/kg·K].

119. Saturated steam enters a condenser at 1 psia and saturated water leaves the condenser. The cooling water enters the condenser tubes at 65 F and leaves at 77 F. Ignore the changes in the kinetic and potential energies and find the steady state entropy production for this fully insulated condenser.

120. A steam turbine operates at a steady state condition with superheated steam entering the turbine at 3 MPa and 450 C. Steam velocity at the inlet is 150 m/s and the inlet steam pipe has a diameter of 0.75 m. After expansion in the turbine, saturated steam leaves the turbine at 100 C and 90 m/s. The power produced by this turbine is 0.32 MW. The turbine is not insulated and heat transfer takes place at an average temperature at the turbine control surface of 225 C. Ignore changes in the potential energy of the steam and find the entropy production rate in the turbine. [Ans.: $\dot{m} = 613.6$ kg/s, $\dot{Q} = -0.0118$ MW, and $\dot{\sigma} \cong 1.91$ MW/K].

121. A cylinder contains 10 kg of air (treated as an ideal gas) at 2 MPa and 365 C. The frictionless piston is held in place by a stop pin. The pin is now removed and the piston is set free to move. Find the special optimum useful work this process. For the surroundings use $P_o = 101$ kPa and 25 C. [Ans.: 3476.2 kJ].

122. A cylinder contains 16 lbm of air at 220 psia. We wish the optimum useful work corresponding with the expansion of a frictionless piston to be 1000 Btu. Find the volume of the tank and air initial temperature to satisfy this requirement. Treat air as an ideal gas and use $P_o = 15$ psia and $T_o = 77$ F.

123. A cylinder contains steam at 460 psia and 600 F. The frictionless piston is held in place by a stop pin. The pin is now removed and the piston is set free to move. After expansion, steam pressure and temperature drop to 100 psia and 360 F while producing 74 Btu/lbm of work and exchanging heat with a sink reservoir at 305 F. Determine the effectiveness of the steam expansion. Use $P_o = 15$ psia and $T_o = 77$ F. [Ans.: 78%].

124. The stored energy of a system containing compressed air can be used in various work processes. Consider a tank containing 5 kg of compressed air at 1.5 MPa and 350 C. Find the maximum useful work. Treat air as an ideal gas and use $P_o = 0.101$ MPa and $T_o = 298$ K. [Ans.: 1635 kJ].

125. We want to compare two methods of heating the same tank. For this purpose, consider increasing the quality of a two-phase mixture in a rigid tank. The tank has a volume of 1 m³. Initially the mixture pressure is 2 MPa and the mixture quality is 8%. The tank temperature is raised to 300 C. The pressure and temperature of the surroundings are 0.101 MPa and 25 C, respectively. In the first method, we fully insulate the tank and use a paddle wheel. In the second method,

we remove the insulation and add heat from a reservoir at 450 C. Find the irreversibility of each method and comment on the result. [Ans.: $I_a = 120,000$ kJ & $I_b = 12,355$ kJ].

126. To maximize the productivity of an electric power plant, we need to determine wasteful processes. Two obvious candidates are the heat carry out of the plant in the heat sink and in the exhaust of stack gases. Use the data and find which stream is more wasteful. The power plant produces 12,000 MWe having an overall efficiency of 31%. The power plant exhausts the stack gases at a rate of 500 lbm/s and a temperature of 445 F. The condenser uses 190,000 lbm/s of cooling water, which enters condenser at 65 F. Treat the stack gases as air and air as an ideal gas. [Ans.: $T_{w2} = 78$ F, $\psi_w = 45,285$ Btu, $\psi_A = 11,294$ Btu].

127. In a heavy duty truck, the circulating water to cool the 600 hp engine enters radiator at 0.2 MPa, 98 C and 3.6 kg/s. Air flows over the radiator tubes at a rate of 8 kg/s. Find the irreversibility of the radiator. Use $T_o = 25$ C and ignore pressure drop in both streams. [Ans.: $T_{w2} = 71.4$ C, $T_{A2} = 78$ C].

128. Find the steady flow special availability of a geothermal energy source. Water from this source is at 0.6 MPa and 152 C.

129. When the combustion products in a diesel engine ignite, temperature reaches 4850 F and 1950 psia. If the combustion products are treated as air, find the associated special availability of the products.

IIb. Power Cycles

Power cycles are an important application of the thermodynamic principles. In this chapter we discuss power cycles that use a heat source to develop a net power output. The heat source may be the energy from fossil fuel, nuclear fuel, solar heating, or geothermal energy.

1. Gas Power Systems

Power production systems using gases as the working fluid have a wide range of applications in automotive, aircraft, and large-scale land-based power plants. The working fluid in such systems always remains in the gas phase throughout a cycle.

1.1. Definition of Terms

Internal combustion engines are power systems in which the working fluid changes composition. Such systems generally use air in addition to the fuel (resulting in combustion products). Examples of such systems include gasoline engines using a spark-ignition system, diesel engines, and gas turbines.

External combustion engines are power systems in which the working fluid does not change composition; rather heat is transferred to the working fluid from the combustion products. An example includes a fossil power plant where heat is transferred to steam, which is the working fluid in a boiler. In nuclear power plants no combustion takes place. Rather, heat is produced by fission and transferred to the coolant. Hence, a nuclear power plant can be simply considered as an external engine or machine. Although external engines generally use steam as working fluid, gas cooled reactors, by definition, are external machines that use a gaseous working fluid to produce work in conjunction with a gas turbine.

Open cycle is a term applied to internal combustion engines because the working fluid changes from cycle to cycle. This occurs, for example, in the intake process of a spark-ignition engine where, air is admitted and mixed with the combustible products. The mixture is then ignited, expanded, and at the end of the cycle the combustion products leave the engine in the exhaust process. The cycle is then repeated.

Reciprocating engines are of the cylinder-piston type. In contrast, a Wankel engine is equipped with a rotor. As discussed in Chapter I, the piston slides inside the cylinder by the connector rod, which is attached to the crankshaft. In a reciprocating engine, depending on the manner the air-fuel mixture enters, the exhaust leaves the cylinder (chamber), and the power stroke per revolution of the crankshaft, the engine may be of a *two-* or a *four-stroke* type.

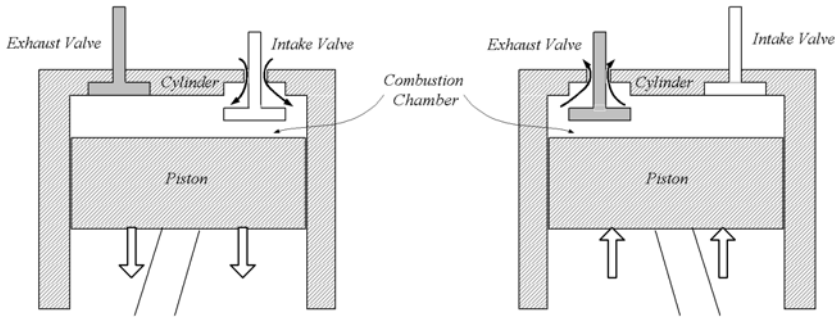


Figure IIb.1.1. Intake and exhaust in a reciprocal open cycle four-stroke internal combustion engine

Shown in Figure IIb.1.1 is a four-stroke open cycle internal combustion engine. The *head-end dead center* and *crank-end dead center* are the positions at which the volume of the combustion chamber is a minimum or a maximum, respectively. At the head-end dead center for example, the piston is fully inserted into the cylinder. In the intake process, the intake valve opens to deliver air from the engine manifold (not shown in Figure IIb.1.1) while the exhaust valve is closed. This condition is reversed when the piston pushes the exhaust gases out of the cylinder.

Figure IIb.1.2 shows the start and the end states of all the processes that constitute one cycle of the operation of an open cycle internal combustion engine. The cycle begins at the *intake* stage when the piston is at the head-end dead center and just begins to move downward to admit the mixture of air and fuel in to the cylinder by suction. At this stage, the exhaust valve is fully closed and the intake valve is fully open. When the piston reaches the crank-end dead center, the intake valve closes. The *compression* process starts at the conclusion of the intake process when the piston begins to move toward the head-end dead center. At the end of the compression process, *ignition* takes place resulting in the *combustion* of the mixture*. This pushes the piston towards the crank-end dead center. Work is delivered in this *expansion* process. Finally, the cycle is completed when the piston moves towards the head-end dead center to discharge the combustion products. In this stage, known as the *exhaust* process, the exhaust valve opens while the intake valve is fully closed. A cylinder-piston engine and the corresponding P - v diagram are shown in Figure IIb.1.3.

* While four-stroke engines have ignition in every other revolution of the crankshaft, two-stroke engines have ignition in every revolution. In two-stroke engines, the lubrication system is eliminated as oil is directly added to the fuel. Thus a two-stroke engine has a higher specific power than a similar four-stroke engine and is used in such appliances as chain saw and leaf blower and in small airplanes. Having ignition per every revolution requires simultaneous compression of the air-fuel-oil mixture while the combustion products are expanding. Similarly, by combining the intake and exhaust processes, the incoming compressed mixture expels the combustion products through the exhaust port. In this process some of the fresh mixture may also escape through the exhaust.

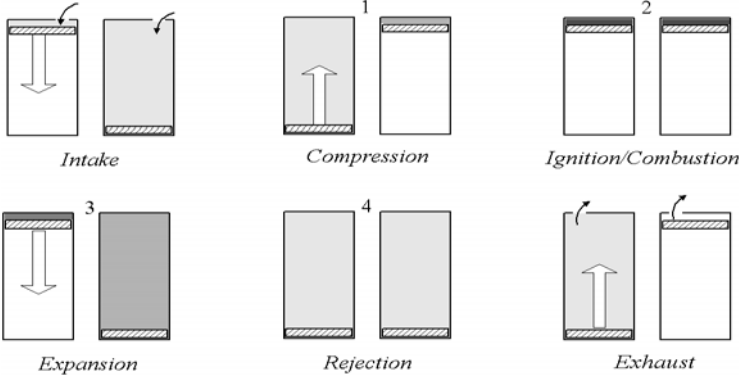


Figure IIb.1.2. Start and end states of processes in an open-cycle internal combustion engine. (Numbers refer to the air standard cycle)

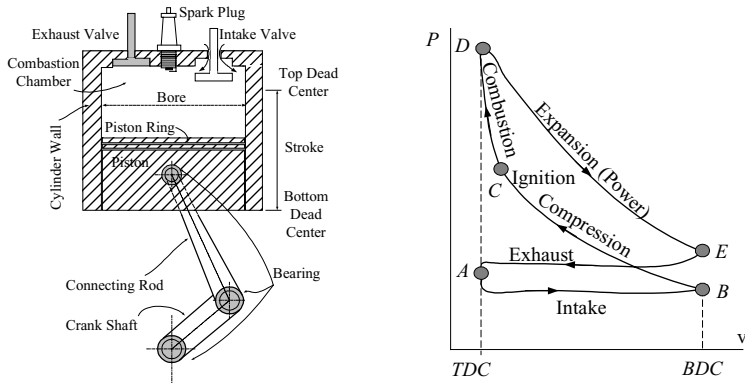


Figure IIb.1.3. A piston-cylinder engine and corresponding pressure-displacement plot

Gas turbine is an internal combustion engine where shaft work is produced by a rotor rather than the moving boundary of a deformable control volume. Air enters at the suction end of a compressor that is driven by the turbine shaft (Figure IIb.1.4). Compressed air leaves the compressor and enters the combustion chamber. This leads to high-energy gases entering the gas turbine. Inside the turbine, the gas flows between the static blades, which act as a diffuser by directing the flow of gas over the rotating blades. The rotating blades are attached to the rotor to transfer momentum and energy. A portion of the work produced by the high-energy gas is delivered to the compressor. The gas finally leaves the turbine by transferring the remaining of energy to the heat sink.

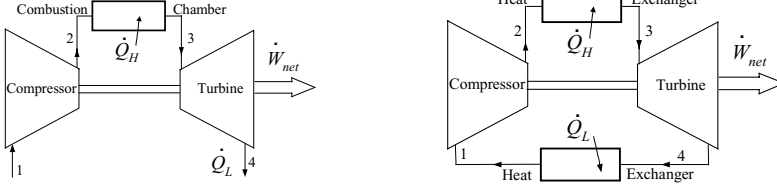


Figure IIb.1.4. Schematics of an open and a closed cycle gas turbine

Closed cycle, also referred to as *air standard cycle*, is a theoretical cycle resembling an actual open cycle for analysis purposes. In an air standard cycle, we make several simplifying assumptions. First, we assume that the working fluid - air - is behaving as an ideal gas. This assumption allows us to use the equation of state for ideal gases and to describe internal energy in terms of the product of temperature and specific heat. Next we assume that the mass of air used as working fluid is fixed in the entire cycle. Hence, there is no intake and no exhaust processes. Heat of ignition is transferred to this fixed mass of air at the heat source and is rejected to the heat sink. Hence, the same mass of air is analyzed throughout a cycle and the composition of air remains intact. We also assume all processes are internally reversible and the effects of kinetic and potential energies are assumed to be negligible.

Compression ratio (r_v) for a process is defined as the ratio of the gas volume before compression to the gas volume after compression, $r_v = V_1/V_2$. Therefore, the compression ratio is always greater than one.

Pressure ratio (r_p) for a process is defined as the ratio of gas pressure after compression to gas pressure before compression, $r_p = P_2/P_1$. Therefore, the pressure ratio is always greater than one. This term is also referred to as the *compressor pressure ratio*. For an isentropic compression, $r_p = r_v^\gamma$.

Temperature ratio (r_T) is defined as the ratio of the maximum to the minimum temperature in a cycle, $r_T = T_3/T_1$.

1.2. Air Standard Carnot Cycle

The air standard Carnot cycle (Figure IIb.1.5) is a theoretical cycle in which heat transfer to air at the heat source and heat rejection at the heat sink take place as isothermal processes while the compression and expansion processes are isentropic. Thus, in the Carnot cycle $s_1 = s_4$, $s_2 = s_3$, $T_1 = T_2 = T_L$, and $T_3 = T_4 = T_H$.

The efficiency for the Carnot cycle is obtained by using Equation IIa.9.1 and substituting for Q_L and Q_H from the T - s diagram of Figure IIb.1.5:

$$\eta_{th} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L(s_1 - s_2)}{T_H(s_4 - s_3)} = 1 - \frac{T_L}{T_H} = \frac{T_1}{T_4} = 1 - \frac{T_2}{T_3} \quad \text{IIb.1.1}$$

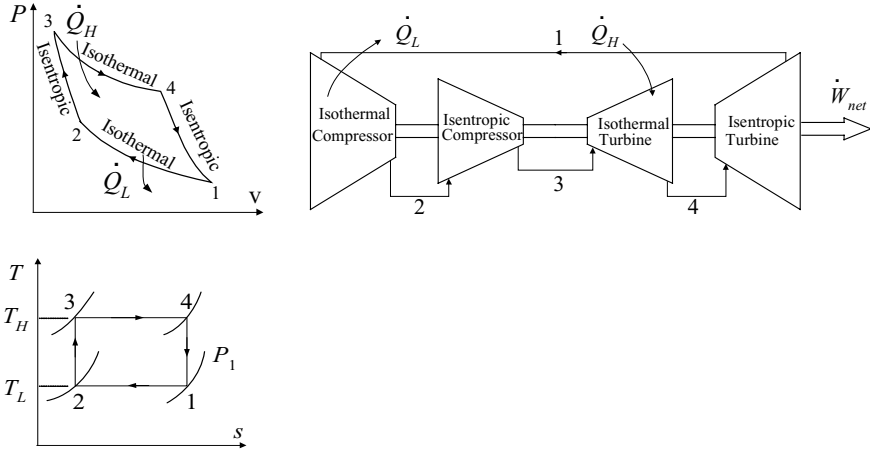


Figure IIb.1.5. The air standard Carnot cycle

For the isentropic process of gas compression we define the compression ratio, r_v as:

$$r_v = \frac{V_1}{V_4} = \frac{V_2}{V_3}$$

Using the relation between pressure and volume for an isentropic process, $PV^\gamma = \text{constant}$ where $\gamma = c_p/c_v$ and the equation of state for an ideal gas, $PV = mRT$, we can relate r_v to temperature ratio as follows:

$$r_v = \frac{V_2}{V_3} = \left(\frac{T_2}{T_3}\right)^{\frac{1}{1-\gamma}} = \left(\frac{P_3}{P_2}\right)^{\frac{1}{\gamma}} \quad \text{IIb.1.2}$$

Since heat transfer to air at the heat source takes place in an isothermal process, we calculate heat from the definition of entropy:

$$Q_{3-4} = \int_3^4 \delta Q = \int_3^4 T ds = T_3(s_4 - s_3)$$

We now use Equation IIa.3.6 with the last term set to zero:

$$s_4 - s_3 = R \ln \frac{V_4}{V_3} + \int_1^2 c_v \frac{dT}{T} = R \ln \frac{V_4}{V_3}$$

Therefore, the amount of heat transferred at the heat source is given as:

$$Q_{34} = T_3 R \ln \frac{V_4}{V_3} = T_3 R \ln \left(\frac{P_3}{P_4} \right)^{\frac{1}{\gamma}} \quad \text{I Ib.1.3}$$

The amount of heat transferred to the heat sink can be found from similar procedure.

Example I Ib.1.1. Find the thermal efficiency of a Carnot cycle for $T_L = 25^\circ\text{C}$ and $T_{H1} = 250^\circ\text{C}$, $T_{H2} = 500^\circ\text{C}$, and $T_{H3} = 750^\circ\text{C}$.

Solution: From Equation I Ib.1.1 $\eta_{th1} = 1 - \frac{T_L}{T_H} = 1 - \frac{25 + 273}{250 + 273} = 1 - 0.57 = 43\%$.

Similarly, for T_{H2} , and T_{H3} we find $\eta_{th1} = 1 - (298/773) = 61\%$ and $\eta_{th3} = 1 - (298/1023) = 70\%$. Thus, for fixed T_L , η_{th} increases with T_H .

In the next problem, we find the important cycle parameters such as pressure, temperature, and the rate of heat transfer.

Example I Ib.1.2. An air standard Carnot cycle operates at an efficiency of 75%. The amount of heat transferred to air at the heat source is 50 Btu/lbm. The highest and the lowest cycle pressures are 2710 psia and 14.7 psia, respectively. Find cycle pressures and temperatures. ($\gamma_{air} = 1.4$).

Solution: We have to find 2 temperatures ($T_1 = T_2$ and $T_3 = T_4$) and 2 Pressures (P_2 and P_4) given Q_{34} , η_{th} , P_1 , and P_3 . To find these unknowns, we use Equations I Ib.8.1, I Ib.8.2, I Ib.8.3, and the isentropic relation. From Equation I Ib.8.1, we have:

$$\eta_{th} = 1 - (T_1 / T_4) = 1 - (P_1 / P_4)^{\frac{1-\gamma}{\gamma}}$$

We find P_4 from here, substitute it into Equation I Ib.8.3, and solve the result for T_3 to get:

$$T_3 = \frac{Q_{34}}{R \ln \left[\frac{P_3}{P_1} (1 - \eta_{th})^{\frac{\gamma}{1-\gamma}} \right]} = \frac{50 \times 778}{53.34 \ln \left[\frac{2710(1 - 0.75)^{3.5}}{14.7} \right]} \approx 2000 \text{ R}$$

Having T_3 , we also have $T_4 = T_3 = 2000 \text{ R}$. We can use Equation I Ib.8.1 again to find other temperatures:

$$\eta_{th} = 1 - \frac{T_2}{T_3} = 0.75$$

Hence, $T_2 = T_1 = 500 \text{ R}$. Having found all temperatures, we can obtain the two remaining pressures. For this purpose we apply the isentropic relation between

states 4 and 1:

$$P_4/P_1 = (T_4/T_1)^{\gamma/(\gamma-1)} = (2000/500)^{3.5} = 128$$

Substituting, we find $P_4 = 14.7 (128) = 1882$ psia. Finally, P_2 is obtained from an isentropic relation between states 2 and 3: $P_2 = (T_2/T_3)^{\gamma/(\gamma-1)} P_3 = 21.2$ psia.

The Carnot cycle is not practical. This is due to the fact that isentropic processes cannot be achieved in practice. Furthermore, heat addition and rejection take place at isothermal processes, which are also difficult to obtain in practice. In theoretical form, the Carnot cycle still serves to predict the maximum efficiency for cycles operating between the same reservoir temperatures. Means of approaching the performance of the Carnot cycle are discussed later in this chapter.

1.3. Air Standard Cycles for Reciprocating Engines

The air standard Otto cycle is the theoretical version of the actual Otto cycle (after Nikolaus August Otto, 1832 – 1891) in spark-ignition reciprocating engines, used extensively in the automotive industry. As shown in Figure I Ib.1.6 (a), the air standard Otto cycle is an isentropic-isochoric cycle where heat addition and rejection take place at constant volume, and compression and expansion processes are at constant entropy.

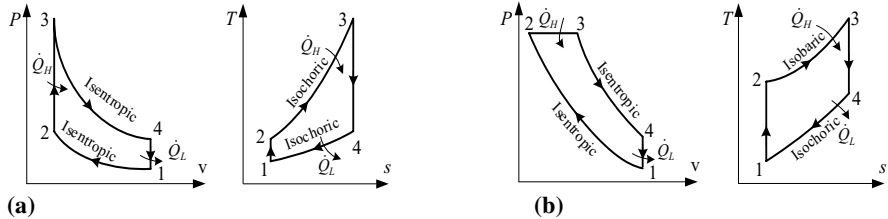


Figure I Ib.1.6. The air standard Pv and Ts diagrams for (a)- Otto and (b)- Diesel Cycles

The efficiency of the Otto cycle is obtained from:

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{\dot{m}c_v(T_4 - T_1)}{\dot{m}c_v(T_3 - T_2)} = 1 - \frac{(T_4/T_1 - 1) T_1}{(T_3/T_2 - 1) T_2} \quad \text{I Ib.1.4}$$

We can manipulate Equation I Ib.1.4 to obtain thermal efficiency only as a function of the pressure ratio. To do this, we take advantage of the isentropic process relating temperature ratio to volume ratio, $T_2/T_1 = (V_1/V_2)^{\gamma-1}$. Finally, we note that the cylinder volume for compression is the same as the volume for expansion ($V_1 = V_4$ and $V_2 = V_3$), hence, $T_2/T_1 = (V_1/V_2)^{\gamma-1} = (V_4/V_3)^{\gamma-1} = T_3/T_4$. Therefore, $T_4/T_1 = T_3/T_2$:

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1} = 1 - \frac{1}{r_V^{\gamma-1}} = 1 - \frac{1}{r_P^{(\gamma-1)/\gamma}} \quad \text{Iib.1.5}$$

This relation shows that theoretically, the larger the compression ratio, the higher the thermal efficiency of the Otto cycle. In practice, however, the larger the compression ratio, the more likely the occurrence of engine knock due to the phenomena known as *detonation*. Combustion is associated with a *flame front* where burning proceeds in combustible gases. Detonation on the other hand is when combustible gases explode rather than burn. The resulting shock waves damage the cylinder, piston, and other engine components. Fuel chemical composition affects the occurrence of detonation in engines with higher compression ratios. For example, leaded fuels reduce the likelihood of detonation hence, allow higher compression ratio. However, the environmental concern regarding lead has resulted in most spark-ignition engines operating at a compression ratio generally in the range of 8 to 9.

Example Iib.1.3. An air standard Otto cycle operates at an efficiency of 55%. The amount of heat transferred to air at the heat source is 1700 kJ/kg. The lowest cycle pressure and temperature are 0.1 MPa and 20 °C, respectively. Find cycle pressures and temperatures at the end of each process.

Solution: We first find the compression ratio from thermal efficiency, Equation Iib.1.5:

$$r_V = \left(\frac{1}{1 - \eta_{th}}\right)^{\frac{1}{\gamma-1}} = \frac{1}{(1 - 0.55)^{2.5}} = 7.36$$

The compression process from 1 to 2 is isentropic for which from Equation Iia.4.4 we have;

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r_V^{\gamma-1} = \frac{1}{1 - \eta_{th}} = 2.22$$

Solving for T_2 we find $T_2 = 2.22 T_1 = 2.22(20 + 273) = 651$ K. We can also find P_2 from:

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \frac{V_1}{V_2} = r_V^{\gamma} r_V = r_V^{\gamma} = (7.36)^{1.4} = 16.36$$

Resulting in $P_2 = 1.636$ MPa. We find T_3 from the following heat balance at the heat source:

$$Q = 1700 = c_v(T_3 - T_2) = 0.7165(T_3 - 651)$$

Hence, we find $T_3 = 3024$ K. Since the process from state 2 to state 3 is isochoric, we find P_3 from

$$P_3 = \frac{T_3}{T_2} P_2 = 3024 \times 1.636 / 651 = 7.6 \text{ MPa}$$

The expansion process from 3 to 4 is isentropic, hence:

$$P_4 = \left(\frac{V_3}{V_4}\right)^\gamma P_3 = \frac{P_3}{r_V^\gamma} = \frac{7.6}{7.36^{1.4}} = 0.465 \text{ MPa}.$$

In the above example, we calculated a gas temperature as high as 3024 K. This is the temperature of the bulk of the gas. The cylinder wall has a temperature much lower than the gas maximum temperature due to the film thermal resistance adjacent to the wall, as discussed in Chapter IVa.

The air standard Diesel cycle is the theoretical version of the actual diesel cycle in reciprocating engines, used extensively in trucking, heavy industry, and irrigation. The diesel cycle (after Dr. Rudolph Christian Karl Diesel, 1858 – 1913) does not have a spark ignition rather it is a compression-ignition cycle. Air is compressed to the ignition temperature of the fuel. As such, diesel engines have a high compression ratio since only air is being compressed. As shown in Figure IIb.1.6, the air standard diesel cycle is an isentropic-isobaric-isochoric cycle where heat is added in an isobaric process and is rejected in an isochoric process. To find thermal efficiency in an air standard diesel cycle, we write:

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{\dot{m}c_v(T_4 - T_1)}{\dot{m}c_p(T_3 - T_2)} = 1 - \frac{(T_4/T_1 - 1) T_1}{\gamma(T_3/T_2 - 1) T_2}$$

IIb.1.6

To simplify Equation IIb.1.6, similar to Equation IIb.1.5, we introduce a new variable called *cutoff ratio* (also referred to as *the degree of isobaric expansion*), $r_c = V_3/V_2$. We now try to find temperature ratios in terms of r_V and r_c . Using the isentropic relations between states 1 and 2, we find

$$P_2 = P_1 r_V^\gamma$$

Using the ideal gas relation between states 2 and 3, noting that $P_2 = P_3$, and taking advantage of the cutoff ratio, we find:

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} \frac{V_3}{V_2} = r_c$$

From the ideal gas relation between states 4 and 1 we find $T_4 = (P_4/P_1)T_1$. To cancel P_4 , we use:

$$\frac{P_4}{P_3} = \left(\frac{V_3}{V_4}\right)^\gamma = \left(\frac{V_3}{V_1}\right)^\gamma = \left(\frac{V_3}{V_2}\right)^\gamma \left(\frac{V_2}{V_1}\right)^\gamma = \left(\frac{r_c}{r_V}\right)^\gamma$$

From the isochoric process between states 1 and 4 we find $T_4/T_1 = P_4/P_1$. Substituting for P_4 in terms of P_3 from the above relation, we obtain:

$$\frac{T_4}{T_1} = \frac{P_3}{P_1} \left(\frac{r_c}{r_V}\right)^\gamma = \frac{P_2}{P_1} \left(\frac{r_c}{r_V}\right)^\gamma = \frac{P_1 r_V^\gamma}{P_1} \left(\frac{r_c}{r_V}\right)^\gamma = r_c^\gamma$$

We need to find similar relations for the remaining temperature ratio. Using the ideal gas relation, we find:

$$\frac{T_1}{T_2} = \frac{P_1}{P_2} \frac{V_1}{V_2} = \frac{1}{r_V^\gamma} r_V = \frac{1}{r_V^{\gamma-1}}$$

Substituting in Equation IIb.1.6, we obtain η_{th} for the diesel cycle:

$$\eta_{th} = 1 - \frac{(T_4/T_1 - 1)}{\gamma(T_3/T_2 - 1)} \frac{T_1}{T_2} = 1 - \frac{r_c^\gamma - 1}{\gamma(r_c - 1)} \frac{1}{r_V^{\gamma-1}} = 1 - \beta \frac{1}{r_V^{\gamma-1}} \quad \text{IIb.1.7}$$

where β is only a function of r_c .

Example IIb.1.4. An air standard Diesel cycle operates at an efficiency of 65% and a compression ratio of 20. The lowest cycle pressure and temperature are 14.5 psia and 70 F, respectively. Find pressures and temperatures of the cycle at the conclusion of each process.

Solution: We first find pressure and temperature of state 2 from state 1 by using the isentropic relations, Equation IIa.4.1:

$$P_2 = P_3 = \left(\frac{V_1}{V_2}\right)^\gamma P_1 = 20^{1.4} \times 14.50 = 961.2 \text{ psia}$$

Similarly,

$$T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} T_1 = r_V^{\gamma-1} T_1 = 20^{0.4} (70 + 460) = 1756.6 \text{ R}$$

Having P_1 , P_2 , P_3 , T_1 , and T_2 , we need to find P_4 , T_3 , and T_4 . These can be found from these 3 equations:

$$\begin{cases} \frac{P_4}{P_1} = \frac{T_4}{T_1} \\ \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} \\ \eta_{th} = 1 - \frac{T_4 - T_1}{\gamma(T_3 - T_2)} \end{cases}$$

To solve this set, we first eliminate P_4 , by substituting it from the first to the second equation. We would then have two equations and two unknowns, T_3 and T_4 . From the first equation we find, $P_4 = (P_1/T_1)T_4$. Substituting into the second equation, we get:

$$T_3 = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{T_1}{T_4} \quad T_4 = \left(\frac{P_2 T_1}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{1}{T_4^\gamma}$$

We now substitute T_3 into the third equation for thermal efficiency and solve for T_4 :

$$T_4 - (1 - \eta_{th}) \gamma \left(\frac{P_2 T_1}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{1}{T_4^\gamma} + [(1 - \eta_{th}) \gamma T_2 - T_1] = 0$$

This is a nonlinear algebraic equation for the unknown T_4 . Substituting for $T_1 = 530$ R, $T_2 = 1756.6$ R, $P_1 = 14.5$ psia, $P_2 = 961.2$ psia, $\gamma = 1.4$, and $\eta_{th} = 0.65$, we get:

$$T_4 - 9.749 T_4^{1/\gamma} + 330.73 = 0$$

The answer can be found by iteration as $T_4 = 1329.5$ R. Back substitution results in $P_4 = (14.5/530)T_4 = 36.4$ psia. Finally, we find $T_3 = (961.2 \times 530/14.5)^{2/7} 1329.5^{1/1.4} = 3388.4$ R. We can find the amount of heat added to the cycle as:

$$q_H = c_p(T_3 - T_2) = 0.24(3388.4 - 1756.6) = 391.6 \text{ Btu/lbm}$$

Similarly,

$$q_L = c_v(T_4 - T_1) = 0.171(1329.5 - 530) = 136.7 \text{ Btu/lbm}$$

$$w_{net} = 392.6 - 136.7 = 254.88 \text{ Btu/lbm}$$

We may check on thermal efficiency as

$$\eta_{th} = 254.88/391.6 = 0.65$$

Note, $(\eta_{th})_{\text{Carnot}} = 1 - (530/3388.4) = 0.84$

1.4. Air Standard Cycle for Gas Turbines

The air standard Brayton cycle* (after George Bailey Brayton, 1830–1892) is an ideal cycle for gas turbine plants as shown in Figure IIb.1.7. Gas compression and expansion takes place in isentropic processes and heat addition and rejection in isobaric processes. By increasing the pressure ratio, the efficiency of the Brayton cycle can be increased. To demonstrate, let's increase the pressure ratio from P_2/P_1 to P_2'/P_1 . This cycle is associated with higher heat addition at the heat source but the same heat transfer to the heat sink as the original cycle hence, higher thermal efficiency. To derive thermal efficiency in terms of the pressure ratio, we start with the definition of thermal efficiency:

* Also known as the Joule cycle.

$$\eta_{th} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{\dot{m} c_p (T_4 - T_1)}{\dot{m} c_p (T_3 - T_2)} = 1 - \frac{(T_4 / T_1 - 1) T_1}{(T_3 / T_2 - 1) T_2}$$

For the isentropic process 3-4 we can write $P_4/P_3 = (V_3/V_4)^k$. Since $P_1 = P_4$ and $P_2 = P_3$, then $V_1/V_2 = V_3/V_4$. From the ideal gas equation of state we have $T_4/T_1 = V_4/V_1$ and $T_3/T_2 = V_3/V_2$. Therefore, $T_4/T_1 = T_3/T_2$. Upon substitution and the application of Equation IIa.4.5, thermal efficiency simplifies to:

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{P_1}{P_2}\right)^{(\gamma-1)/\gamma} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} \quad \text{IIb.1.8}$$

where in Equation IIb.1.8, we also made use of the definition of r_p , the pressure ratio.

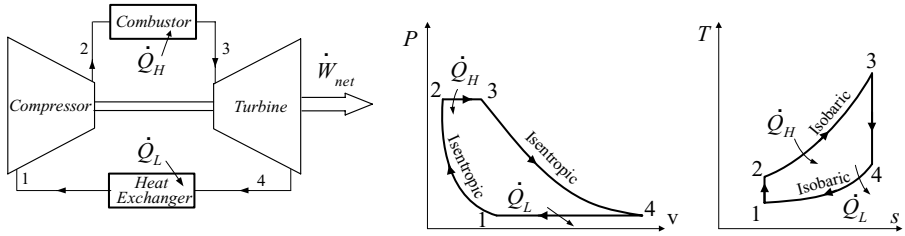


Figure IIb.1.7. The air standard P - v and T - s diagrams for Brayton cycle

Maximum work in the air standard Brayton cycle is a function of the pressure and temperature ratio. To derive this function we start with the following relations for a Brayton cycle as obtained from Equation IIa.4.5:

$$\frac{T_2}{T_1} = r_p^\alpha, \quad \frac{T_4}{T_3} = \frac{1}{r_p^\alpha}$$

where in these relations $\alpha = (\gamma - 1)/\gamma$. The net work per unit mass of the Brayton cycle is found as:

$$\begin{aligned} w_{net} &= w_t - w_c = c_p (T_3 - T_4) - c_p (T_2 - T_1) = c_p T_1 \left[\left(1 - \frac{T_4}{T_3}\right) \frac{T_3}{T_1} - \left(\frac{T_2}{T_1} - 1\right) \right] \\ &= c_p T_1 \left[\left(1 - \frac{1}{r_p^\alpha}\right) r_T - \left(r_p^\alpha - 1\right) \right] \end{aligned}$$

Where w_t and w_c are work per unit mass delivered by the turbine and work per unit mass delivered to the compressor, respectively. We also substituted for the temperature ratios in terms of r_p .

For a given temperature ratio, we optimize the net work by taking its derivative with respect to r_p and setting the result equal to zero:

$$\frac{\partial w_{net}}{\partial r_p} = c_p T_1 \left[r_T \frac{\partial}{\partial r_p} \left(1 - \frac{1}{r_p^\alpha} \right) - \frac{\partial}{\partial r_p} (1 - r_p^\alpha) \right] = 0$$

After simplification, we find;

$$r_p = (r_T)^{\frac{\gamma}{2(\gamma-1)}} \quad \text{I Ib.1.9}$$

where in Equation I Ib.1.9, $r_T = T_3/T_1$. The maximum net work occurs when r_p is given by Equation I Ib.1.9 since the second derivative of w_{net} is positive.

Example I Ib.1.5. Air enters the compressor of an air standard Brayton cycle at 15°C and 0.1 MPa. The cycle pressure and temperature ratios are given as $r_p = 11$ and $r_T = 5$, respectively. Find:

- pressure and temperature at the end of each process,
- thermal efficiency,
- work per unit mass,
- cycle pressure ratio for optimum work,
- the required mass flow rate to produce 10 MW power.

Solution: a) The inlet temperature to the compressor is $T_1 = 15 + 273 = 288$ K. Air pressure at the exit of the compressor is $P_2 = 0.1 r_p = 0.1 \times 11 = 1.1$ MPa. Hence, $T_2 = T_1 (r_p)^{(\gamma-1)/\gamma} = (15 + 273)(11)^{(1.4-1)/1.4} = 571$ K. Air pressure at the inlet to the turbine is $P_3 = P_2 = 1.1$ MPa and $T_3 = T_1 (r_T) = 288 \times 5 = 1440$ K. Finally, $P_4 = P_1 = 0.1$ MPa and $T_4 = T_3 (1/r_p)^{(\gamma-1)/\gamma} = 1440/(11)^{1.4-1/1.4} = 726$ K.

b) Thermal efficiency is:

$$\eta_{th} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} = 1 - 1/(11)^{0.4/1.4} = 49.6\%$$

c) Work per unit mass is found from:

$$w_{net} = c_p (T_3 - T_4) - c_p (T_2 - T_1) = 1.0035(1449 - 726 - 571 + 288) = 432.5 \text{ kJ/kg}$$

d) The optimum r_p is found from Equation I Ib.8.9 as $r_p = (5)^{1.4/[2(1.4-1)]} = 16.72$. This is a high compression ratio due to a high temperature ratio. For a temperature ratio of 4, the pressure ratio is found to be 11.3. Therefore, in the above example with pressure ratio of 11, if the temperature ratio is maintained at 4, the cycle would produce near maximum work. This is important for transportation applications of the gas turbine to maximize net work per unit mass of the working fluid. This, in turn, is related to *specific power* (power produced by the engine divided by the weight of the engine).

e) The relationship between power and net work per unit mass is:

$$\dot{W}_{net} = \dot{m}w_{net}$$

where \dot{m} is the mass flow rate of the working fluid in the cycle. For the air standard cycle, the mass flow rate of air needed to produce 10 MW of power is found as $\dot{m} = 10,000 \text{ kW} / 432.5 \text{ kJ/kg} = 23 \text{ kg/s}$.

Gas turbine with regenerator is designed to recover some energy from the hot exhaust gases. A counter flow heat exchanger (regenerator), uses the turbine exhaust gases to heat up the compressed gases before they enter the turbine. Heating the gas from state 2 to state “a” before entering the combustor saves fuel.

According to the second law of thermodynamics, $T_a \leq T_4$ (Figure IIb.1.8). This is expressed in terms of the regenerator efficiency:

$$\eta_{reg} = \frac{T_a - T_2}{T_4 - T_2}$$

Generally, regenerator efficiency ranges from 60 to 80%. While addition of the regenerator helps to save fuel, it also introduces an additional initial investment.

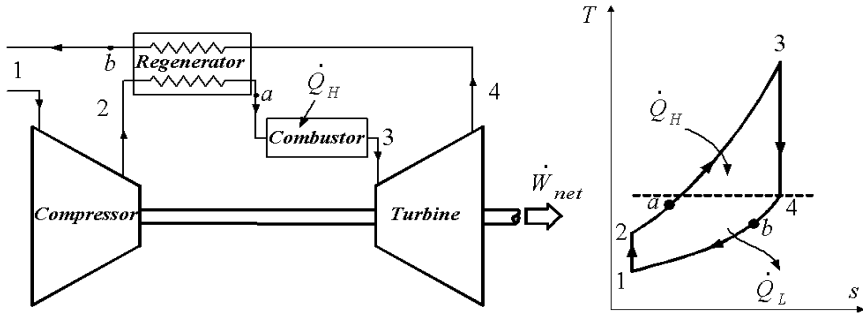


Figure IIb.1.8. A Regenerative modified Brayton Cycle

Example IIb.1.6. Consider the addition of a regenerative heat exchanger with an efficiency of 75% to the cycle of Example IIb.1.5. Find the improvement in thermal efficiency.

Solution: The same $w_{net} = 432.5 \text{ kJ/kg}$ as in Example IIb.1.5 also applies here. However, the amount of heat addition at the heat source is reduced due to the addition of the regenerator. To calculate the amount of heat addition, we first find T_a from:

$$T_a = T_2 + (T_4 - T_2)\eta_{reg} = 571 + 0.75(726 - 571) = 687 \text{ K}$$

Total heat addition is:

$$Q_H = c_p(T_3 - T_a) = 1.0035(1440 - 687) = 755 \text{ kJ/kg}$$

Thermal efficiency becomes:

$$\eta_{th} = \frac{w_{net}}{Q_H} = \frac{432.5}{755} = 57\%$$

The improvement in thermal efficiency is in excess of 15%, justifying the addition of the regenerator.

Air standard Stirling and Ericsson cycles are examples of how to approach the Carnot efficiency in common practice. One impractical aspect of the Carnot cycle is the fact that the heat addition and rejection are isothermal processes. The Stirling cycle (after Rev. Robert Stirling, 1790–1878) and Ericsson cycle (after Captain John Ericsson, 1803–1889) can be approximated if the heat addition and rejection processes take place in multiple stages. Heating and cooling of gas in the Stirling cycle take place in isochoric processes and in the Ericsson cycle in isobaric processes. As shown in Figure I Ib.1.9, these cycles can achieve the thermal efficiency of the Carnot cycle operating within the same T_H and T_L temperature limits.

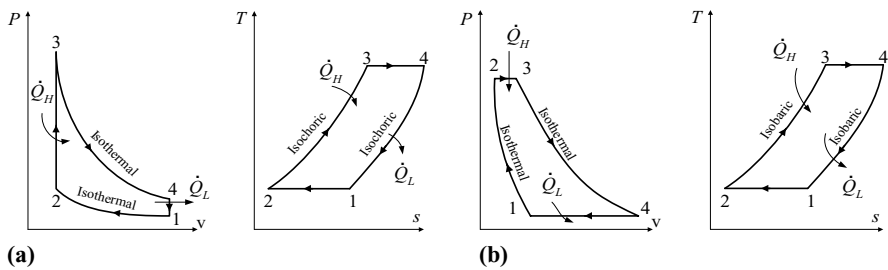


Figure I Ib.1.9. The air standard Pv and Ts diagrams for (a) Stirling and (b) Ericsson cycles

Approaching Ericsson and Stirling cycles is possible by devising systems for heating and cooling in multiple stages so that the average temperature at each process represent the intended temperature of the theoretical Stirling and Ericsson cycles (Figure I Ib.1.10). Systems allowing multiple stage heat addition and heat rejection are called reheat and intercooler, respectively.

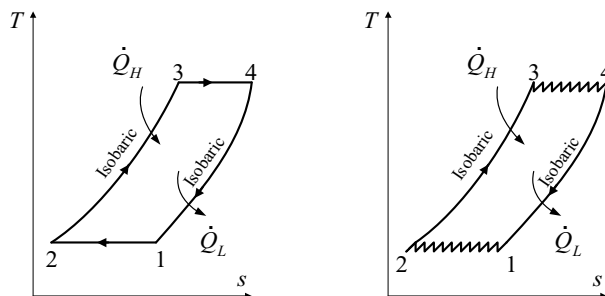


Figure I Ib.1.10. Ericsson and the approximate Ericsson cycle

Gas turbine with intercooler is shown in Figure IIb.1.11. Addition of the intercooler increases the net work of the gas turbine cycle. A cycle equipped with an intercooler compresses the working fluid at two stages. The compressed gas at the exit of the first stage is cooled in a heat exchanger and compressed to the intended pressure in the second stage. The use of an intercooler increases net work hence, the cycle thermal efficiency, but there is a limit to the number of stages that can be added to the cycle due the associated cost and diminishing gain.

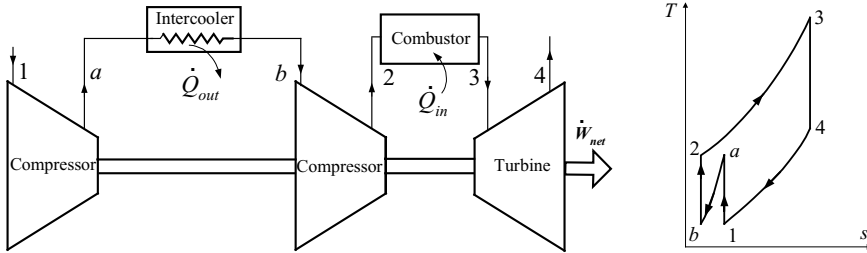


Figure IIb.1.11. Gas turbine equipped with intercooler

Gas turbine with reheat allows for gas expansion in multiple stages. Hot gases enter the first stage of the turbine and are heated up to the same temperature before entering the second stage of the turbine (Figure IIb.1.12). The reheat increases net work of the cycle, hence, the cycle efficiency.

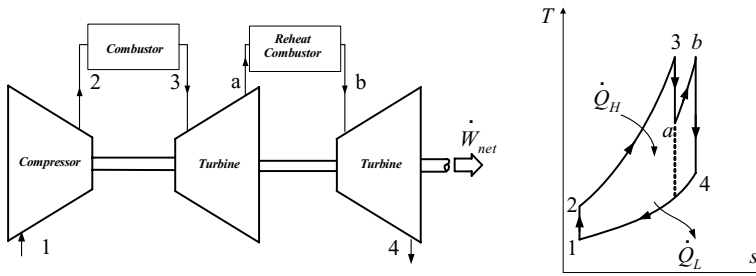


Figure IIb.1.12. The reheat modified Brayton Cycle

1.5. Air Standard Cycle for Reaction Engines

Reaction engines are gas turbines where the bulk of the work developed by the turbine is converted into the kinetic energy and used as thrust for propulsion. Towards the end of World War II, aircraft equipped with reciprocating engines could reach a maximum speed of up to 500 miles/hour. The advent of aircraft with reaction engines increased the maximum speed to over 1000 miles/hour. This was possible because reaction engines have a much higher specific power compared with reciprocating engines.

Gas turbine for jet propulsion¹ is shown in Figure I Ib.1.13. Air pressure is increased in a diffuser before entering the compressor. The work produced by the turbine is primarily delivered to the compressor and the remaining power is used as auxiliary power for lighting, air conditioning, and other electrical needs of the aircraft. The exhaust gases from the turbine are expanded to the atmospheric pressure in a nozzle. The change in the momentum due to this expansion produces the force required to thrust the aircraft.

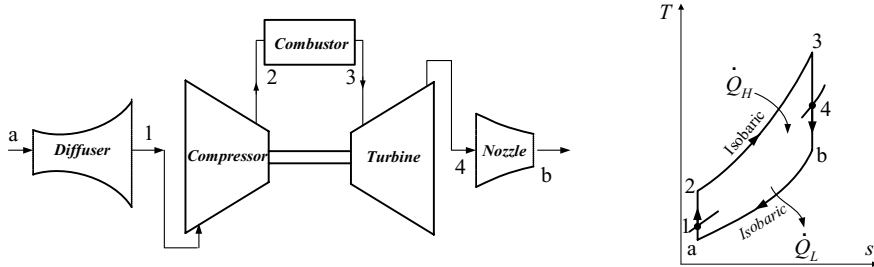


Figure I Ib.1.13. Schematic of a gas turbine for jet propulsion²

Example I Ib.1.7. Air enters a jet engine diffuser at 650 miles/h (1046 km/h), 5 F (-15 C) and 12 psia (83 kPa). The compression ratio of the compressor is 10. The gas temperature at the exit of the combustor is 1700 F (927 C). If all processes are ideal and turbine work is delivered entirely to the compressor, find the velocity of gases at the nozzle exit. The diffuser inlet diameter is 6 ft (1.83 m).

Solution: We first determine properties at the end of various processes. For this, we ignore all potential and kinetic energies except for the nozzle. Having $P_a = 12$ psia, $T_a = 5 + 460 = 465$ R (258 K), and $V_a = 650$ miles/hr = 953.3 ft/s (290.5 m/s) we find properties at state 1 from the first law of thermodynamics:

$$h_1 = h_a + \frac{V_a^2}{2} = 0.24 \times 465 + \frac{953.3^2}{2 \times 32.2 \times 778} = 129.7 \text{ Btu/lbm} \quad (301.7 \text{ kJ/kg})$$

Therefore $T_1 = h_1/c_p = 540.5$ R. To find P_1 , we use the isentropic process for the diffuser:

$$P_1 = P_a \left(\frac{T_1}{T_a} \right)^{\gamma/(\gamma-1)} = 12 (540.5/465)^{0.4/1.4} = 12.5 \text{ psia} \quad (86.2 \text{ kPa})$$

¹ This topic is discussed in more details in Chapter VIc.

² Sir Frank Whittle (1907–1996) and Hans J. P. von Ohain independently developed jet airplane engine. Whittle obtained his jet-propulsion patent in 1930. The first British experimental jet flew in 1941.

Having P_1 , we find P_2 from the compression ratio; $P_2 = 10 \times 12.5 = 125$ psia. To find T_2 , we use the isentropic relation for the compression process;

$$T_2 = T_1 r^{\gamma-1/\gamma} = 540.5(10)^{0.4/1.4} = 1043.5 \text{ R } (579.4 \text{ K})$$

Since process 2–3 is an isobaric process, $P_3 = P_2 = 125$ psia and T_3 is given as $T_3 = 1700 + 460 = 2160$ R. To find the state of air at the turbine exit, we note that $w_T = w_C$. If written in terms of enthalpies we find:

$$h_2 - h_1 = h_3 - h_4$$

Treating air as an ideal gas, which allows us to use a constant specific heat, we find that:

$$T_4 = T_3 + T_1 - T_2 = 2160 + 540.5 - 1043.5 = 1657 \text{ R } (920.2 \text{ K})$$

Air pressure at state 4 can be found from the isentropic relation written for the expansion process in turbine:

$$P_4 = P_3 \left(\frac{T_4}{T_3} \right)^{\gamma/(\gamma-1)} = 125(1657/2160)^{0.4/1.4} = 115.9 \text{ psia } (0.8 \text{ MPa})$$

We can also find temperature at state 5 from the isentropic expansion in the nozzle where air reaches the atmospheric pressure, $P_b = P_a$:

$$T_b = T_4 \left(\frac{P_b}{P_4} \right)^{(\gamma-1)/\gamma} = 1657(12/115.9)^{0.4/1.4} = 866.8 \text{ R } (499 \text{ K})$$

We can find velocity at the nozzle exit by writing the energy equation for a control volume encompassing the nozzle:

$$V_b = \sqrt{2c_p(T_4 - T_b)} = \sqrt{2 \times 0.24 \times 32.2 \times 778(1657 - 866.8)} = 3082.5 \text{ ft/s } (939.65 \text{ m/s})$$

The thrust developed by various types of gas turbines for aircraft propulsion is discussed in Chapter VIc.

2. Vapor Power Systems

Unlike the gas power systems in which the working fluid is constantly changing, the vapor power cycles use a closed system in which the working fluid remains the same but its phase changes during a cycle. The vapor power systems are primarily based on the Rankine cycle. To improve thermal efficiency, the Rankine cycle is modified with reheat and regenerative cycles. In the calculations, we use the first and the second law of thermodynamics in conjunction with the steam tables thermodynamics properties.

2.1. Definition of Terms

Rankine and modified Rankine cycle are extensively used in electric power plants using steam as working fluid.

Balance of plant is a term generally applied to include all the components in a power plant except the heat source. This includes turbine, condenser, pump, feedwater heater, and the associated piping.

Feedwater is the water flowing from the condenser to the heat source.

Extraction steam is a term applied to that portion of steam that bypasses the turbine to heat up feedwater.

Feedwater heater is a heat exchanger used to heat up feedwater from the extraction steam to increase η_{th} .

High-, intermediate-, and low-pressure turbines are stages of a steam turbine that admit steam at progressively decreasing pressures.

Reheater is a heat exchanger to heat up the steam exiting the high-pressure turbine prior to entering the intermediate-pressure turbine. The warmer stream is extraction steam from the heat source that bypasses the high-pressure turbine.

Moisture separator transfers the condensate of the reheater to a tank to be pumped to the feedwater line. The tank is known as the drain tank and the pump as the drain pump.

Trap is a valve that reduces steam pressure by introducing a large, non-recoverable pressure drop to the flow and allows the condensate to pass to a lower pressure region. Pressure drop is discussed in Chapter IIb.

2.2. The Rankine Cycle

A schematic of a Rankine cycle (after William John Maquorn Rankine, 1820 – 1872) used for a steam power plant is shown in Figure IIb.2.1. Water is pumped isentropically into the heat source at state 1. The heat source can be a boiler, the vessel of a BWR, the steam generator of a PWR, etc. Water is boiled at constant pressure and the saturated or superheated steam enters the high-pressure stage of the steam turbine. The stationary blades direct high-energy steam toward the rotating blades on the turbine shaft, which then turns the rotor of the electric generator. In the Rankine cycle, the steam expansion process in the turbine is isentropic ($s_3 = s_4$). The low-energy steam leaves the turbine at stage 4 and enters the condenser. After rejecting heat in the heat sink at constant pressure, it is again pumped into the heat source for the next cycle.

Thermal efficiency of a Rankine cycle is calculated from:

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{W}_t - \dot{W}_p}{\dot{Q}_H} = \frac{\dot{m}(h_3 - h_4) - \dot{m}(h_2 - h_1)}{\dot{m}(h_3 - h_2)} \quad \text{IIb.2.1}$$

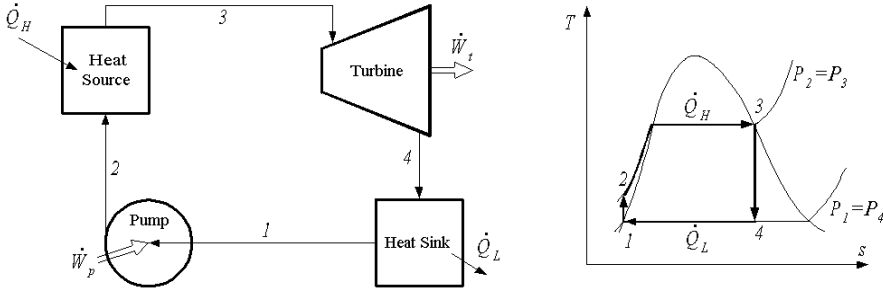


Figure Iib.2.1. Rankine cycle for a steam power plant

where subscripts p and t stand for pump and turbine, respectively. Recall that, per Equation Iia.6.7, the enthalpy rise through the pump is found as $\Delta h_{pump} \cong v_f(T)\Delta P_{pump}$. Substituting into Equation Iib.2.1, thermal efficiency becomes:

$$\eta_{th} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = \frac{(h_3 - h_4) - v_f(T)\Delta P_{pump}}{(h_3 - h_1) - v_f(T)\Delta P_{pump}} \quad \text{Iib.2.2}$$

Example Iib.2.1. Saturated steam in a Rankine cycle enters a turbine at 5.86 MPa (850 psia) and leaves the condenser at 6.895 kPa (1 psia). Find the cycle thermal efficiency.

Solution: We first find the relevant thermodynamic properties at the heat sink and heat source pressures:

P (MPa)	v_f (m ³ /kg)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kg·K)	s_g (kJ/kg·K)
0.00689	0.10074E-2	162.178	2571.07	0.5552	8.2791
5.86	—	1205.44	2785.44	3.0125	5.8998

The energy used in pumping the condensate is found from:

$$w_p = v_f(T)\Delta P_{pump} = 0.10074\text{E-}2 \times (5.86\text{E}3 - 0.00689\text{E}3) = 5.89 \text{ kJ/kg}$$

Therefore, $h_2 = h_1 + w_p = 162.178 + 5.89 = 168 \text{ kJ/kg}$. To find η_{th} , we need h_1 through h_4 (Equation Iib.2.2).

We have $h_1 = 162.178 \text{ kJ/kg}$, $h_2 = 168 \text{ kJ/kg}$, and $h_3 = 2785.44 \text{ kJ/kg}$. To find h_4 , we first find x_4 from the second law of thermodynamics. Process 3-4 is isentropic

From Equation IIa.10.14 we conclude that, $s_3 = s_4$:

$$x_4 = \frac{s_3 - s_{f4}}{s_{g4} - s_{f4}} = \frac{5.8998 - 0.5552}{8.2791 - 0.5552} = 0.692$$

We can now find $h_4 = 162.178 + 0.692(2571.07 - 162.178) = 1829.13$ kJ/kg. Thus thermal efficiency is:

$$\eta_{th} = \frac{w_{net}}{\dot{q}} = \frac{(2785.44 - 1829.13) - 5.89}{(2785.44 - 162.178) - 5.89} = \frac{950.42}{2617.37} = 36.3\%$$

As discussed in Example IIb.1.5, the total power produced by a power plant is the product of flow rate and the net work per unit mass of the working fluid.

Example IIb.2.2. In Example IIb.2.1, find the steam mass flow rate for a 1000 MW power plant.

Solution: The net power per unit mass flow rate is found from $w_{net} = w_t - w_p = (h_3 - h_4) - w_p$. Substituting, the net work becomes $w_{net} = 950.42$ kJ/kg. Hence, the required steam mass flow rate to produce 1000 MW power is $\dot{m}_s = \dot{W} / w_{net} = 1\text{E}6 / 950.42 = 1052$ kg/s (8.35×10^6 lbm/h).

Effects of pressure and temperature on cycle performance. We now investigate the effects of lowering heat sink pressure, superheating steam, and increasing steam pressure on the cycle thermal efficiency. Given heat source pressure and temperature, lowering heat sink pressure increases cycle efficiency. To verify, we use the Rankine cycle in Figure IIb.2.2. Lowering heat sink pressure for the same amount of heat addition, causes the amount of heat rejection to be reduced by area 1-4-4'-1'-2'-2-1 in Figure IIb.2.2(a). On the other hand, lowering the heat sink pressure results in $x_{4'} < x_4$. This decrease in steam quality is disadvantageous for the turbine blades as excessive moisture would lead to pitting and erosion. For a given heat sink pressure, cycle efficiency increases by superheating steam. This is shown in Figure IIb.2.2(b) where work is increased by the enclosed area in 3-3'-4'-4-3. This amount of extra work is obtained at the expense of more heat input in the heat source shown by the enclosed area in 3-3'-a'-a. Further advantage of superheating steam is the increase in steam quality leaving the last stage of the turbine. Finally, given heat sink pressure and steam temperature, increasing heat source pressure increases cycle efficiency but reduces steam quality leaving the last stage of the turbine, Figure IIb.2.2(c). To remedy this problem, we use a modified Rankine cycle by reheating steam, as discussed later in this chapter. In the following example we investigate the effect of steam superheat on cycle efficiency.

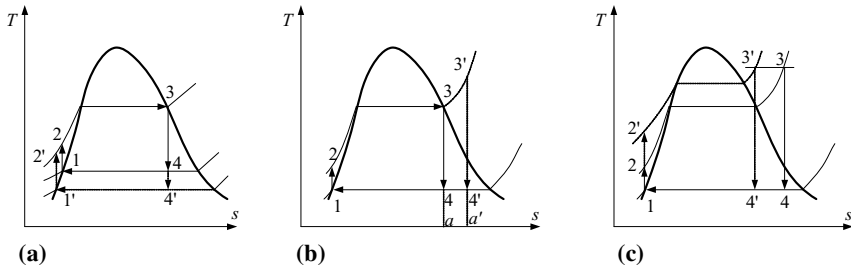


Figure IIB.2.2. Pressure and temperature effects on Rankine cycle

Example IIB.2.3. In Example IIB.2.1, instead of saturated steam, suppose we use superheated steam at a temperature of 650 F (616 K). The rest of the cycle remains unchanged. Find the effect on cycle efficiency.

Solution: We find the relevant thermodynamic properties at the given heat source pressure and temperature as well as the given heat sink pressure. Saturation properties at the heat sink pressure remain the same:

P (psia)	v_f (ft ³ /lbm)	h_f (Btu/lbm)	h_g (Btu/lbm)	s_f (Btu/lbm·F)	s_g (Btu/lbm·F)
1.000	0.016136	69.730	1105.8	0.1326	1.9781

Superheated properties at heat source pressure and temperature are:

P (psia)	T (F)	v (ft ³ /lbm)	h (Btu/lbm)	s (Btu/lbm·F)
850.0	680	0.71250	1323	1.5283

We note that the pump work remains unchanged. Thermal efficiency is found from Equation IIB.2.2. We have $h_1 = 69.73$, $h_2 = 72.27$, and $h_3 = 1198$ Btu/lbm. To find h_4 , we first find x_4 :

$$x_4 = \frac{s_3 - s_f}{s_g - s_f} = \frac{1.5283 - 0.1326}{1.9781 - 0.1326} = 0.76$$

Therefore, $h_4 = 69.73 + 0.76 (1105.8 - 69.73) = 857$ Btu/lbm. Thus, thermal efficiency becomes:

$$\eta_{th} = \frac{(h_3 - h_4) - v_f(T)\Delta P_{pump}}{(h_3 - h_1) - v_f(T)\Delta P_{pump}} = \frac{(1323 - 857.0) - 2.54}{(1323 - 69.73) - 2.54} = 37\%$$

In the above example, we were only interested in finding the improvement in the thermal efficiency. For the sake of completion, it is important to also calculate

such key design parameters as turbine work, the energy deposited in the heat source, and the energy rejected to the environment in the heat sink.

Example I Ib.2.4. In Example I Ib.2.3, find the heat added in the heat source and rejected in the heat sink.

Solution: To find the heat added to the working fluid, we write an energy balance for the heat source:

$$q_H = h_3 - h_2 = 1323 - 72.27 = 1250.73 \text{ Btu/lbm (2909 kJ/kg)}$$

The amount of energy rejected to the surroundings is also found from an energy balance written for the heat sink:

$$q_L = h_4 - h_1 = 857 - 69.73 = 787.27 \text{ Btu/lbm (1831 kJ/kg)}$$

The net work is $w_{net} = q_H - q_L = 1250.73 - 787.27 = 463.5 \text{ Btu/lbm}$. Thermal efficiency can be found from $\eta_{th} = w_{net} / q_H = 463.5 / 1250.73 = 37\%$. Turbine work is $w_t = h_3 - h_4 = 1323 - 857 = 466 \text{ Btu/lbm (1804 kJ/kg)}$. Finally, $w_{net} = w_t - w_p = 466 - 2.54 = 461.75 \text{ Btu/lbm (1074 kJ/kg)}$.

To numerically verify the effect of the heat source and heat sink pressures on thermal efficiency, we may perform a parametric study the results of which are shown in Figure I Ib.2.3.

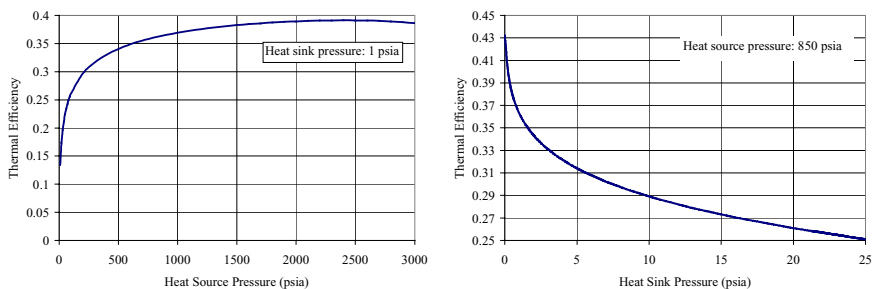


Figure I Ib.2.3. Effect of pressure on thermal efficiency of an ideal Rankine cycle

Figure I Ib.2.3 indicates that, for a given heat sink pressure, thermal efficiency increases with increasing heat source pressure. Conversely, for a given heat source pressure, thermal efficiency decreases substantially with increasing heat sink pressure.

2.3. Reheat-Modified Rankine Cycle

As discussed above, increasing heat source pressure or decreasing heat sink pressure increases thermal efficiency. However, such changes in pressure also increase the moisture content in the last stage of the turbine, Figures I Ib.2.2(a) and I Ib.2.2(c). The reheat cycle helps alleviate the high moisture content, as

shown in Figure IIb.2.4. After expansion to some intermediate pressure, steam is heated up in an isobaric process before entering the next stage of the turbine. By doing so, we increase steam quality from $x_{6'}$ to x_6 .

To accomplish the same goal and also increase thermal efficiency, we could have increased the degree of superheat to state $3'$ (Figure IIb.2.4).

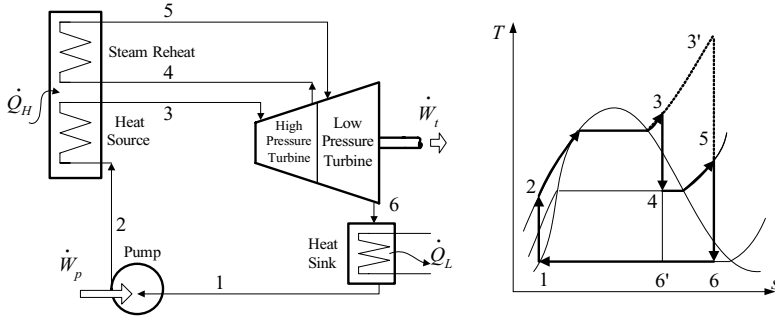


Figure IIb.2.4. The reheat-modified Rankine cycle

2.4. Regenerative-Modified Rankine Cycle

In this modification, feedwater is heated by steam extraction from the turbine prior to entering the heat source. This increases the average temperature in the heat source, thereby increasing thermal efficiency. Heating up of water takes place in a heat exchanger referred to as the *feedwater heater* (FWH). In such systems, steam condenses on the tubes carrying the feedwater. Hence, the two streams do not mix and are generally at different pressures. Since the two streams do not mix, these are known as *closed feedwater heaters*. Occasionally, streams may be allowed to mix, which takes place when the two streams are at the same pressure. This is referred to as *open feedwater heaters*, as shown in Figure IIb.2.6(a) and discussed in Example IIa.7.5. In closed feedwater heaters, the condensate is either pumped to a higher-pressure FWH, Figure IIb.2.6(b), or allowed to flow to a lower pressure region, such as either a FWH or the condenser, Figure IIb.2.6(c). In the latter case, the condensate is passed through a special valve referred to as steam trap. Ideally, the mixture pressure in the steam trap drops in an isentropic process to the pressure of the up-stream system. Such a system is either a low pressure FWH or the condenser.

To determine the fraction of steam extraction from the turbine to be used in an open feedwater heater so that state 3 is saturated liquid, we use an energy balance written for the feedwater heater. From Figure IIb.2.5, for perfect mixing, we have:

$$\dot{m}_{es}h_6 + (\dot{m}_s - \dot{m}_{es})h_2 = \dot{m}_sh_3$$

where \dot{m}_s and \dot{m}_{es} are mass flow rates of steam and the extraction steam, respectively. Dividing terms by the steam mass flow rate and showing the fraction of steam used as extraction steam by y , we get:

$$yh_6 + (1 - y)h_2 = h_3$$

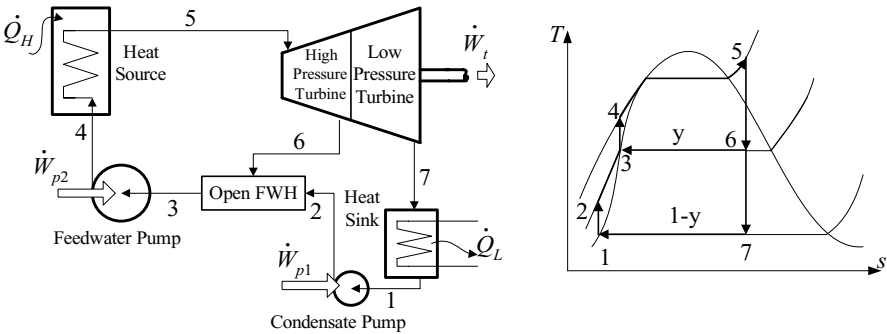


Figure I Ib.2.5. Regenerative-modified Rankine cycle. Open feedwater heater.

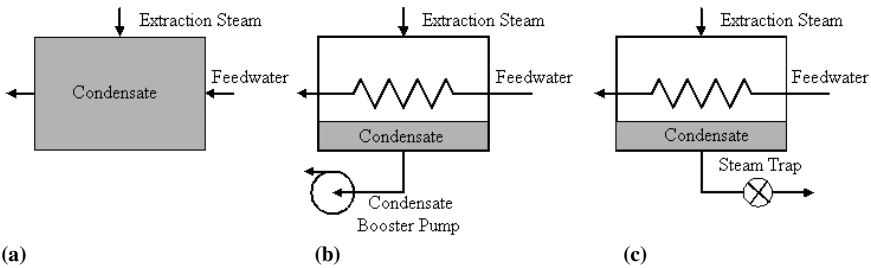


Figure I Ib.2.6. Schematics of open and closed feedwater heaters

Example I Ib.2.5. In Example I Ib.2.1, we now introduce an open feedwater heater with steam extraction to heat up the feedwater. For the FWH operating at 100 psia (0.7 MPa), find the revised thermal efficiency.

Solution: We first find the relevant thermodynamic properties at the given pressures.

P (psia)	v_f (ft ³ /lbm)	h_f (Btu/lbm)	h_g (Btu/lbm)	s_f (Btu/lbm·F)	s_g (Btu/lbm·F)
1.000	0.016136	69.730	1105.8	0.1326	1.9781
100.0	0.017740	298.50	1187.2	0.4743	1.6027
850.0	0.021500	518.40	1198.0	—	1.4096

To find steam quality at state 6, we use $s_6 = s_5$ (unlike Figure I Ib.2.5, here state 5 is saturated), hence:

$$1.4096 = 0.4743 + x_6 (1.6027 - 0.4743)$$

From here, $x_6 = 0.83$ and $h_6 = 298.5 + 0.83(1187.2 - 298.5) = 1036.1$ Btu/lbm (2410 kJ/kg). The energy used in the condensate pump, which delivers the con-

densate to the FWH is found from:

$$w_{cp} = v_f(T)\Delta P_{pump} = 0.016695 \times (100 - 1) \times (144 / 778) = 0.31 \text{ Btu/lbm} \quad (0.72 \text{ kJ/kg})$$

where subscript *cp* stands for condensate pump. Therefore, $h_2 = h_1 + 0.31 = 70$ Btu/lbm (163 kJ/kg). We can find y , the fraction of steam used as steam extraction, from an energy balance for the FWH:

$$yh_6 + (1 - y)h_2 = h_3$$

$$1036.1y + 70(1 - y) = 298.5$$

Therefore, $y = 0.236$. Having the fraction of steam used for steam extraction, we can calculate w_t and w_p . To do this, we first find the enthalpy of state 4. The energy used in the feedwater pump is found from:

$$w_{fwp} = v_f(T)\Delta P_{pump} = 0.01774 \times (850 - 100) \times (144 / 778) = 2.46 \text{ Btu/lbm} \quad (5.72 \text{ kJ/kg})$$

where subscript *fwp* stands for feedwater pump. Therefore, $h_4 = h_3 + w_p$. Finding $h_4 = 298.50 + 2.46 = 301$ Btu/lbm. Total pumping power is:

$$w_p = (1 - y)w_{cp} + w_{fwp} = (1 - 0.236) \times 0.31 + 2.46 = 2.69 \text{ Btu/lbm} \quad (6.26 \text{ kJ/kg})$$

Total power produced by the turbine is:

$$w_t = h_5 - [yh_6 + (1 - y)h_7] = 1198 - [0.236 \times 1036.1 + (1 - 0.236) \times 786.7] = 352.4 \text{ Btu/lbm} \quad (819.7 \text{ kJ/kg})$$

Total energy input is:

$$q_H = h_5 - h_4 = 1198 - 301 = 897 \text{ Btu/lbm} \quad (2086 \text{ kJ/kg})$$

The cycle thermal efficiency is, therefore,

$$\eta_{th} = (w_t - w_p) / q_H = (352.4 - 2.69) / 897 = 0.389.$$

This is an improvement of over 6%. To maximize thermal efficiency, we can find an optimum pressure for the FWH by trial as discussed later in this chapter. There is an initial investment for the reheat and regenerative modifications that will be recovered due to higher efficiency. Also note that w_n has dropped 14% from 408.7 to 349.7 Btu/lbm.

Regenerative Cycle with Closed Feedwater Heater

In general, steam power plants use closed feedwater heaters, as shown in Figure II.2.7. To determine the fraction of steam extraction from turbine to be used in the feedwater heater, we use an energy balance written for the feedwater heater to obtain:

$$y = \frac{(h_3 - h_2)}{(h_5 - h_7)}$$

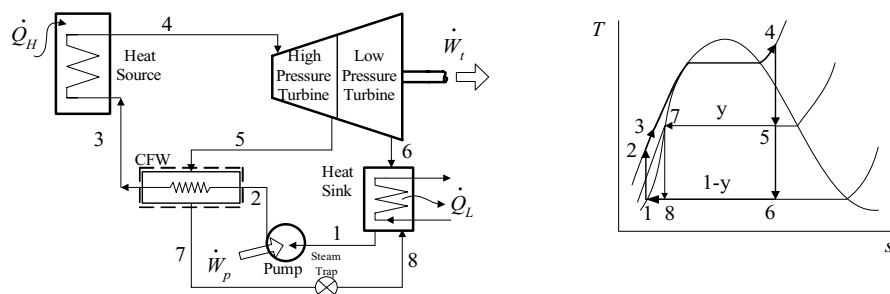


Figure I Ib.2.7. Regenerative-modified Rankine cycle. Closed feedwater heater

The calculation procedure is similar to that of the open feedwater heater as shown in the next example.

Example I Ib.2.6. In Example I Ib.2.5, we use a closed feedwater heater and steam extraction to heat up the feedwater. The extraction steam enters the FWH at 80 psia (0.55 MPa), find thermal efficiency.

Solution: We first find the relevant thermodynamics properties at the given pressures.

P (psia)	T (F)	v_f (ft ³ /lbm)	h_f (Btu/lbm)	h_g (Btu/lbm)	s_f (Btu/lbm·F)	s_g (Btu/lbm·F)
1.000	101.74	0.016136	69.730	1105.8	0.1326	1.9781
80.0	312.04	0.017573	282.10	1183.0	0.4534	1.6208
850.0	525.24	0.021500	518.40	1198.0	—	1.4096

To find steam quality at state 5, we use $s_5 = s_4$ (unlike Figure II.9.7, here steam entering the turbine is saturated), hence:

$$1.4096 = 0.4534 + x_5 (1.6208 - 0.4534)$$

From here, $x_5 = 0.82$ and $h_5 = 282.1 + 0.82(1183.0 - 282.1) = 1020$ Btu/lbm. The energy used in the condensate pump, which delivers the condensate to the FWH is $w_p = 2.54$ Btu/lbm. Hence, $h_2 = 72.27$ Btu/lbm. To find the fraction of steam used as steam extraction we ignore the temperature difference and assume that $T_3 \approx T_5 = 312.04$ F. Having $P = 850$ psia and $T = 312.04$, $h_3 \approx 283.5$ Btu/lbm. Hence

$$y = (283.5 - 72.27)/(1020 - 282.1) = 0.286$$

Turbine work is found from $w_t = (h_4 - h_5) + (1 - y)(h_5 - h_6) = (1198 - 1020) + 0.714(1020 - 786.7) = 344.5$ Btu/lbm. Also $q_H = h_4 - h_3 = 1198 - 283.5 = 914.5$ Btu/lbm. Hence, $\eta_{th} = (344.4 - 2.54)/914.5 = 0.374$

Regenerative Cycle with Moisture Separation

Steam cycles generally employ a moisture separator for steam extraction from the high-pressure turbine, as shown in Figure IIb.2.8. This would introduce dry, saturated steam to the next stage of the turbine. This also reduces the moisture content of steam at the last stage of the low-pressure turbine. For given P_H , P_M , P_L , and h_5 , we can design the cycle if either state 3 is specified or y is given. Note that h_6 is obtained from the isentropic expansion in the high pressure turbine ($s_6 = s_5$) and the first law of thermodynamics written for the moisture separator. Next, we find h_3 or y from an energy balance written for the open feedwater heater:

$$\dot{m}_f h_A + y \dot{m}_g h_7 + (1 - y) \dot{m}_g h_2 = \dot{m}_s h_3 \quad \text{IIb.2.3}$$

where we assumed perfect mixing in the open feedwater heater. Subscripts f and g stand for saturated water and steam flowing out of the moisture separator, respectively. Subscript s stands for the total steam flowing out of the heat source so that $\dot{m}_f + \dot{m}_g = \dot{m}_s$, $\dot{m}_f = (1 - x_6) \dot{m}_s$ and $\dot{m}_g = x_6 \dot{m}_s$.

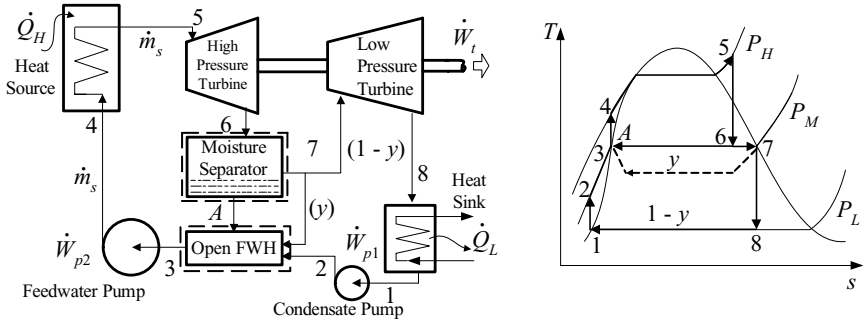
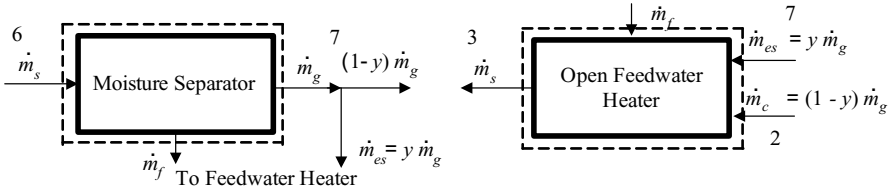


Figure IIb.2.8. A regenerative-modified ideal Rankine cycle with moisture separator



We find h_3 from Equation IIb.2.3 for given y or solve for y if state 3 is specified. If y is given, h_3 is found from Equation IIb.2.3 as $h_3 = x_6 h_A + x_6 y h_7 + x_6 (1 - y) h_2$. If h_3 is given, for example, state 3 is saturated water at pressure P_M (i.e., $h_3 = h_A$ on Figure IIb.2.8) then y is found from Equation IIb.2.3 as:

$$y = (h_3 - h_2) / (h_7 - h_2) \quad \text{IIb.2.4}$$

Example I Ib.2.7. Saturated steam enters a turbine at 1125 psia (7.75 MPa). Heat is rejected in the condenser at 1 psia (6.895 kPa). Find η_{th} for a regenerative cycle using a moisture separator and an open feedwater heater at 50 psia (0.34 MPa). In this case, water leaving the feedwater heater is saturated ($h_3 = h_A$).

Solution: We first find the relevant thermodynamic properties at the given pressures.

P (psia)	v_f (ft ³ /lbm)	h_f (Btu/lbm)	h_g (Btu/lbm)	s_f (Btu/lbm·F)	s_g (Btu/lbm·F)
1.000	0.016136	69.730	1105.8	0.1326	1.9781
50.00	0.017274	250.20	1174.1	0.4112	1.6586
1125.	—	—	1188.0	—	1.37655

We now find the unknown enthalpies in successive steps as follows:

$$w_{p1} = v_f(T) \Delta P_{pump} = 0.016136 \times (50 - 1) \times (144 / 778) = 0.146 \text{ Btu/lbm} \quad (0.34 \text{ kJ/kg})$$

$$h_2 = 69.73 + 0.146 = 69.88 \text{ Btu/lbm} \quad (162.5 \text{ kJ/kg})$$

$$w_{p2} = v_f(T) \Delta P_{pump} = 0.017274 \times (1125 - 50) \times (144 / 778) = 3.44 \text{ Btu/lbm} \quad (8 \text{ kJ/kg})$$

$$h_4 = 250.2 + 3.437 = 253.64 \text{ Btu/lbm} \quad (590 \text{ kJ/kg})$$

$$x_6 = (1.37655 - 0.4112) / (1.6586 - .4112) = 0.774$$

$$h_6 = 250.2 + 0.774(1174.1 - 250.2) = 965.2 \text{ Btu/lbm} \quad (2245 \text{ kJ/kg})$$

$$x_8 = (1.6586 - 0.1326) / (1.9781 - .1326) = 0.827$$

$$h_8 = 69.73 + 0.827(1105.8 - 69.73) = 926.43 \text{ Btu/lbm} \quad (2155 \text{ kJ/kg})$$

Having h_3 , we now find the fraction of the total steam used as extraction steam from Equation I Ib.2.4: $y = (250.2 - 69.88) / (1174.1 - 69.88) = 0.163$

Having all enthalpies and the steam extraction fraction, we can calculate the pump work, the turbine work, the net work, the heat addition, and thermal efficiency in successive steps as follows:

$$w_p = (\dot{W}_{p1} + \dot{W}_{p2}) / \dot{m}_s = x_6(1 - y)w_{p1} + w_{p2} = 0.774 \times (1 - 0.163) \times 0.146 + 3.44 = 3.53 \text{ Btu/lbm} \quad (8.2 \text{ kJ/kg})$$

$$w_t = (h_5 - h_6) + x(1 - y)(h_7 - h_8) = (1188 - 965.2) + 0.774(1 - 0.163)(1174.1 - 926.43) = 383.25 \quad (891 \text{ kJ/kg})$$

$$w_n = w_t - w_p = 383.25 - 3.53 = 379.72 \text{ Btu/lbm} \quad (883 \text{ kJ/kg})$$

$$q_H = h_5 - h_4 = 1188 - 253.64 = 934.36 \text{ Btu/lbm} \quad (2173 \text{ kJ/kg})$$

$$\eta_{th} = 379.72 / 934.36 = 0.406$$

Selection of pressure at which a feedwater heater is operating is not arbitrary. As shown in Figure I Ib.2.10, a regenerative-modified Rankine cycle using an open FWH and a moisture separator while operating between $P_H = 1000$ psia, and $P_L = 1$ psia, has a maximum thermal efficiency at a FWH pressure of about 100 psia.

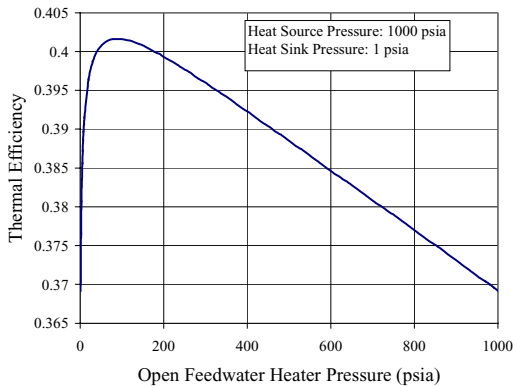


Figure IIb.2.9. Effect of the FWH pressure (P_M) on thermal efficiency

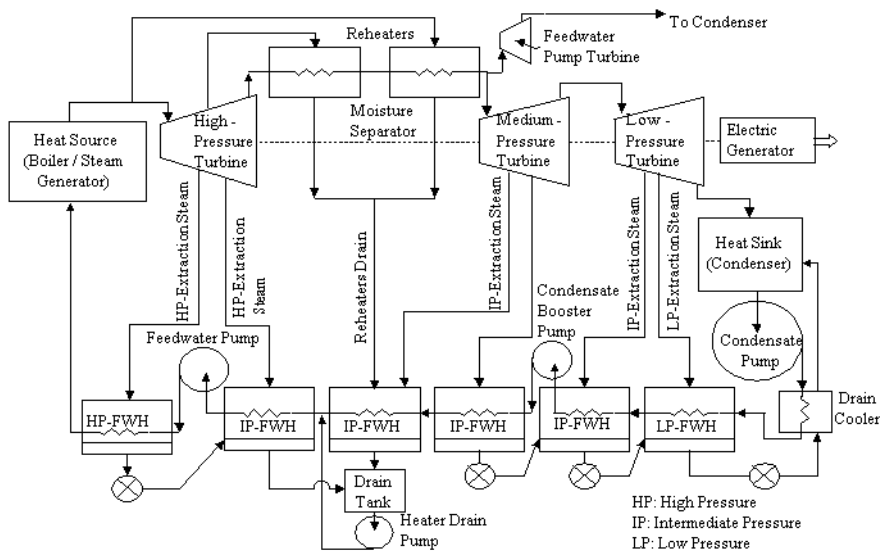


Figure IIb.2.10. A steam power plant utilizing multiple feedwater heaters

In steam power plants, several stages of feedwater heaters operating at various pressures (Figure IIB.2.10) are used. This requires at least four sets of pumps including the condensate pump, the condensate booster pump, the heater drain pump, and the feedwater pump. The condensate pump takes suction from the condenser and delivers water to the first stage of the low pressure FWHs via an external drain cooler. The cooler has two functions. First, it subcools the FWH drain water to prevent flashing in the drain line. Second, it preheats the incoming condensate water before being exposed to the higher energy extraction steam. The feedwater enters the first set of the low-pressure feedwater heaters (LP-FWH) after flowing through the drain cooler. The condensate booster pump takes suction

from the LP-FWH to discharge the feedwater to the intermediate-pressure feedwater heaters. Finally, the feedwater pump delivers water through the high-pressure feedwater heater (HP-FWH) to the heat source. The condensed steam in the secondary side of the IP-FWH and HP-FWH is collected in the *heater drain tank* to be pumped into the feedwater line by the *heater drain pump* also known as the *drip pump*.

In steam power plants, thermal hydraulic loads are generally divided between sets of pumps, coolers, and feedwater heaters to facilitate maintenance and increase system reliability. As shown in Figure I Ib.2.11, three condensate pumps, three feedwater booster pumps, and two feedwater pumps are used to deliver water through 3 coolers and 14 feedwater heaters.

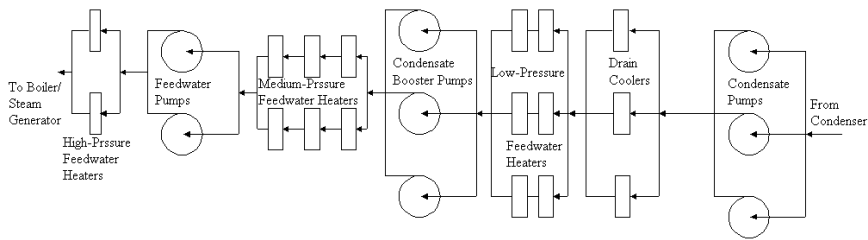


Figure I Ib.2.11. Schematics of a typical steam plant feedwater

3. Actual Versus Ideal Cycles

In the discussion about the vapor and the gas power cycles, we assumed ideal conditions for the involved processes. For example, in the steam power cycles we used the same pressure and temperature for the steam entering the turbine as that leaving the heat source. In reality however, there are pipelines carrying steam from the heat source to the turbine. This is associated with some heat loss even though the pipe is well insulated. Additionally, the flow of steam in the pipe causes a frictional pressure drop. Such non-isentropic effects would adversely affect thermal efficiency.

3.1. Losses in Mechanical Components

In the discussion below, we consider non-isentropic conditions, which result in losses in pipes, turbines, pumps/compressors, and nozzles. In this discussion we consider a steam power plant that utilizes the Rankine cycle as shown in Figure I Ib.3.1. We first consider the pump and turbine losses, then losses in the piping and the condenser.

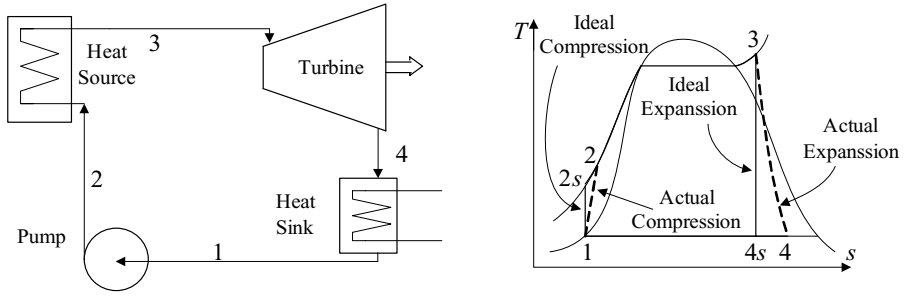


Figure IIb.3.1. Losses associated with pump and turbine

Pump losses are primarily associated with the irreversibilities due to friction between the flow of water and the interior pump surfaces. As shown in Figure IIb.3.1, if the compression process was isentropic, the state at the pump exit would have been $2s$. However, irreversibilities cause entropy to increase and the actual state is 2. To facilitate analysis, we calculate work delivered to the pump by multiplying the isentropic work by the pump efficiency, a value smaller than unity:

$$w_p = w_{ps}\eta_p = h_2 - h_1 = (h_{2s} - h_1)\eta_p$$

Turbine losses are similar to the pump losses and are primarily associated with irreversibilities due to the friction between the flow of steam and the interior turbine surfaces such as the stationary and moving turbine blades. Since there are some heat losses to the environment, the expansion process is non-adiabatic. As shown in Figure IIb.3.1, if the expansion process was isentropic, the state at the turbine exit would have been $4s$. However, irreversibilities cause entropy to increase and the actual state is 4. Interestingly, this decreases the moisture content, which is the only helpful aspect of turbine irreversibilities. To facilitate analysis, we then calculate work obtained from the turbine by multiplying the isentropic work by turbine efficiency, being a value smaller than unity:

$$w_t = w_{ts}\eta_t = h_3 - h_4 = (h_3 - h_{4s})\eta_t$$

Losses in pipes include the losses due to friction as well the heat transfer to the surroundings. These losses are further discussed in Chapter IIIb where they are divided into two categories of *major* and *form* losses. The major loss (skin friction) accounts for friction between the working fluid and the pipe wall. Form loss accounts for the existence of such pipe fitting as elbows, tees, reducers, and valves.

Effects of pipe, pump, and turbine losses on the ideal Rankine cycle are shown in Figure IIb.3.2. Water is first pumped from the condenser to the heat source. If the compression process was isentropic, the state of fluid at the entrance to the heat source would have been at $2s$. However, due to pump irreversibilities, the state of water at the pump discharge is 2. From point 2 to point 3, which is the heat source inlet, pressure and temperature drop due to losses in the pipe ($P_3 < P_2$).

Then water is boiled and superheated steam leaves the heat source at $P_4 = P_3$. Due to pipe losses, superheated steam enters the turbine at $T_5 < T_4$ and $P_5 < P_4$. In this discussion, we ignored losses in the heat source and the heat sink.

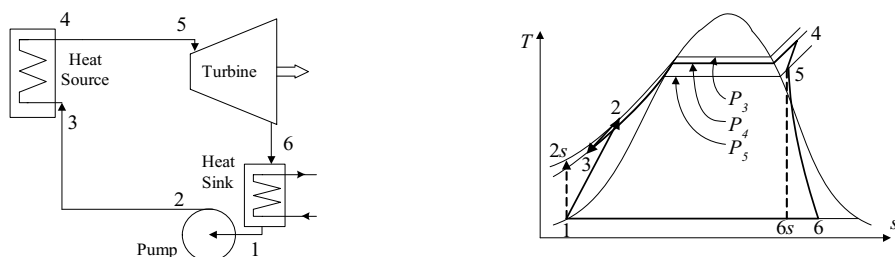
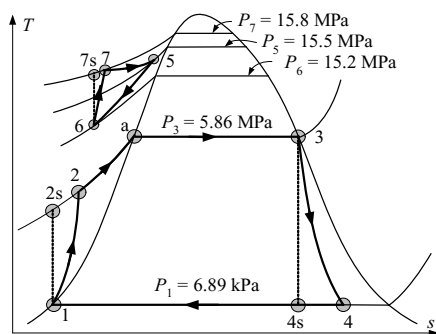
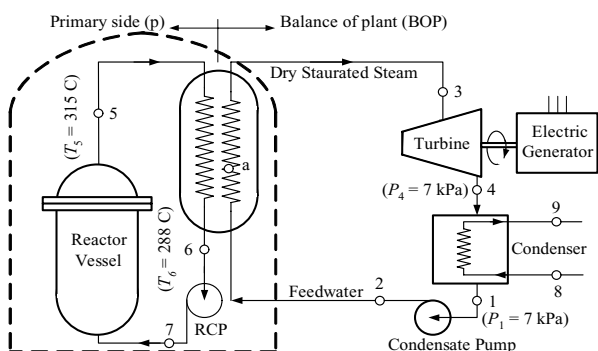


Figure I Ib.3.2. Effect of pipe, pump, and turbine losses on the ideal Rankine cycle

Example I Ib.3.1. The top figure shows the schematics of the primary side of a PWR and the Rankine cycle of the secondary side. The bottom figure shows the associated T - s diagram for both primary and secondary sides. Find a) thermal efficiency, and b) the power produced by the plant. Additional data: mass flow rate in vessel: 60E6 kg/h, $\eta_t = 0.90$, and $\eta_p = 0.85$.



Solution: The ideal cycle is solved in Example Iib.2.1 and the following enthalpies are obtained:

$$h_1 = 162.178 \text{ kJ/kg}, h_{2s} = 168 \text{ kJ/kg}, h_3 = 2785.44 \text{ kJ/kg}, \text{ and } h_{4s} = 1829.13 \text{ kJ/kg}.$$

Due to irreversibilities, the pumping power is increased and turbine work is decreased. To find the revised w_p and w_t , we use the specified isentropic efficiencies:

$$w_p = (h_{2s} - h_1)/\eta_p = (168 - 162.178)/0.85 = 6.85 \text{ kJ/kg}.$$

$$\text{We also find } h_2 = 162.178 + 6.85 = 169.03 \text{ kJ/kg}$$

$$w_t = (h_3 - h_{4s})\eta_t = (2785.44 - 1829.13) \times 0.90 = 860.68 \text{ kJ/kg}$$

$$q_H = h_3 - h_2 = 2785.44 - 169.03 = 2616.4 \text{ kJ/kg}$$

a) We find $\eta_{th} = (w_t - w_p)/q_H = (860.68 - 6.85)/2616.4 = 32.6\%$ (versus 36.3 in Example Iib.2.1)

b) To find the power produced, we use an energy balance in the steam generator:

$$\dot{m}_p(h_5 - h_6) = \dot{m}_s(h_3 - h_2).$$

We need h_5 and h_6 . These are found as $h_5(15.5 \text{ MPa} \ \& \ 315 \text{ C}) = 1421.94 \text{ kJ/kg}$ and $h_6(15.2 \text{ MPa} \ \& \ 288 \text{ C}) = 1273.24 \text{ kJ/kg}$. Therefore,

$$\dot{m}_s = 60\text{E}6 \times (1421.94 - 1273.24)/2616.4 = 3.41\text{E}6 \text{ kg/h}.$$

$$\dot{W} = 3.41\text{E}6 \times (860.68 - 6.85) = 808.7 \text{ MW}$$

QUESTIONS

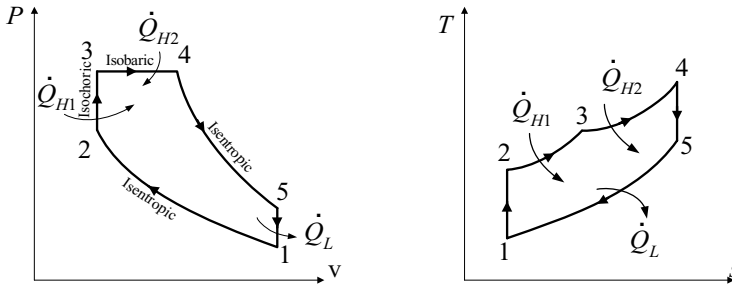
- Would a machine gun be considered an internal combustion or an external combustion engine?
- In an automotive engine, what are the BDC and TDC?
- Is the Wankel engine a reciprocating engine?
- In an internal combustion engine, where is the position of the piston during the rejection process?
- What is the key advantage of a gas turbine over traditional piston-cylinder engine for aviation?
- What is compression ratio?
- During which process in an Otto cycle does heat rejection take place?
- During which process in a Diesel cycle does heat addition take process?
- What is the ultimate heat sink for an automotive engine?
- Why does an increase in heat source pressure of an ideal Rankine cycle increase the cycle thermal efficiency? Assume heat sink pressure and steam temperature are held constant.
- What is the major difference between the Ericsson and the Stirling cycles?
- What is the advantage of an intercooler in a gas turbine?
- By the use of what processes can we approach the Stirling and Ericsson cycles?
- During which process does heat addition take place in a Rankine cycle?
- Why should dry steam enter a turbine?

- What is a reheat-modified Rankine cycle?
- What is a regenerative-modified Rankine cycle?
- What is a moisture separator?
- Explain the effect of feedwater heater pressure on cycle thermal efficiency in a regenerative modified ideal Rankine engine which also uses a moisture separator.

PROBLEMS

1. Consider two cycles. One cycle for a gas turbine and the other cycle for a vapor power plant. Assume that the two cycles have the same power output from the turbine per unit mass flow rate (\dot{W}_t / \dot{m}). Compare the compression work per unit mass flow rate of the gas turbine cycle (\dot{W}_c / \dot{m}) with that of the vapor power cycle (\dot{W}_p / \dot{m}). What conclusion do you reach? Explain the result.
2. An air standard Otto cycle operates at a compression ratio of 4 and a pressure ratio (P_3/P_2) of 4. Find the cycle thermal efficiency for $P_1 = 1$ bar and $T_1 = 320$ K.
3. The compression and the pressure ratios of an Otto cycle are both equal to 4. Air enters the engine at 1 bar and 320 K. Find all pressures and temperatures of this cycle. [Ans.: $P_2 = 7.4$ bar, $P_3 = 29.6$ bar, $P_4 = 4.2$ bar, $T_2 = 592$ K, $T_3 = 368$ K, $T_4 = 1340$ K].
4. An air standard diesel cycle has an efficiency of 0.58 and a compression ratio of 17. Determine pressures and temperatures of the cycle at the conclusion of each process. Pressure and temperature at the start of the compression process are 0.1 MPa and 16 C, respectively. [Ans.: $P_2 = 765.6$ Psia, $P_4 = 67.23$ psia, $T_2 = 1615$ R, $T_3 = 4830$ F, $T_4 = 2411$ R.]
5. An air standard Diesel cycle operates at a compression ratio of 20 and a cutoff ratio of 2. Find the cycle thermal efficiency for $P_1 = 1$ bar and $T_1 = 320$ K.
6. An air standard diesel cycle has a compression ratio of 20 and an isobaric expansion ratio (V_3/V_2) of 2. Pressure and temperature at the start of the compression process are $P_1 = 1$ bar and $T_1 = 350$ K. Find the cycle thermal efficiency. Compare the result with the Carnot efficiency. [Ans.: $\eta = 64.8\%$]
7. Given the same compression ratio for both air standard Otto and air standard Diesel cycle, which cycle has higher thermal efficiency? Answer the same question this time for the Otto cycle versus the Brayton cycle.
8. In this problem we are asked to perform a parametric study for thermal efficiency of an air standard Diesel cycle as a function of the cutoff ratio, r_c , and the compression ratio, r . Use Equation Iib.1.7 and plot thermal efficiency for $r_c = 0.5, 1, 2, 4, 6$, and 8 while the compression ratio is held constant at $r = 10$. Repeat this for $r = 12, 15, 17$, and 20. What conclusion can be reached from this plot? [Ans.: Thermal efficiency increases as r increases and decreases as r_c increases.]

9. Consider the dual cycle shown in the figure. Find thermal efficiency in terms of γ , r , r_P , and r_c . Where $\gamma = c_p/c_v$, $r_V = V_1/V_2$, $r_P = P_3/P_2$, and $r_c = V_4/V_3$.



$$[\text{Ans.: } \eta_{th} = 1 - \frac{r_P r_c^\gamma - 1}{[(r_P - 1) + \gamma r_P (r_c - 1)]} \frac{1}{r_V^{\gamma-1}}].$$

10. Use the following information for a dual cycle internal combustion engine and find a) the state parameters (P & T) at stages 1 through 5 shown in the Problem 9 figure, b) the expansion work, c) the compression work, d) the net cycle work, and the cycle efficiency. Data: $P_1 = 1$ bar, $T_1 = 300$ K, $r_V = 12.7$, $r_P = P_3/P_2 = 1.4$, and $r_c = 1.6$, the working fluid is air ($c_v = 0.72$ kJ/kg·C, $c_p = 1.01$ kJ/kg·C, $R = 287$ J/kg·C, $\gamma = 1.4$). [Ans.: $P_2 = 35$ bar, $P_3 = 49$ bar, $P_4 = 49$ bar, $P_5 = 2.7$ bar, $T_2 = 555$ C, $T_3 = 887$ C, $T_4 = 1585$ C, $T_5 = 535$ C)].

11. Find thermal efficiency of an air standard Brayton cycle with the ratio $T_4/T_3 = 0.45$. [Ans.: 55%]

12. Air enters an ideal gas turbine cycle at 1 bar and 27 C. The maximum temperature in the cycle is 727 C. Find the cycle efficiency for a compression ratio of 10. [Ans.: 0.482].

13. Air enters the compressor of an air standard Brayton cycle at 27 C and 0.1 MPa. The cycle pressure and temperature ratios are given as $r_P = 11$ and $r_T = 4$, respectively. Find a) pressure and temperature at the end of each process, b) thermal efficiency, c) work per unit mass, d) cycle pressure ratio for optimum work, and e) the required mass flow rate to produce 1 MW power.

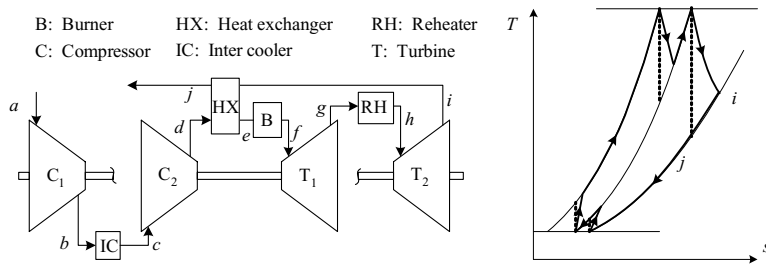
14. Air enters the compressor of an air standard Brayton cycle at 70 F and 14.7 psia. The cycle pressure ratio is $r_P = 11$ and the temperature ratio is such that the net work produced by the cycle corresponds to the maximum net work. Find a) pressure and temperature at the end of each process, b) thermal efficiency, and c) the required mass flow rate to produce 300 horsepower.

15. To improve thermal efficiency, a gas turbine uses a two-stage compression and one intercooler. Assume ideal processes and isentropic compression to show that the compressor work is at a minimum when the two compressors have identical compression ratios. Gas enters both compressors at the same temperature.

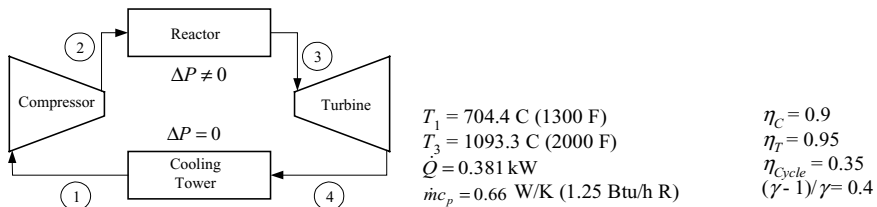
16. In a steam power plant, saturated steam at a pressure of 400 psia enters the turbine. The condenser pressure is atmospheric. Find the ideal Rankine cycle and the maximum thermal efficiency. [Ans.: 23% and 26%].

17. Derive a relation for the optimum r_p of a gas turbine operating on a Brayton cycle having the turbine and the compressor efficiencies of η_t and η_c , respectively. [Ans.: $r_p = (\eta_c \eta_t r_T)^\beta$ where $\beta = \gamma/2(\gamma - 1)$].

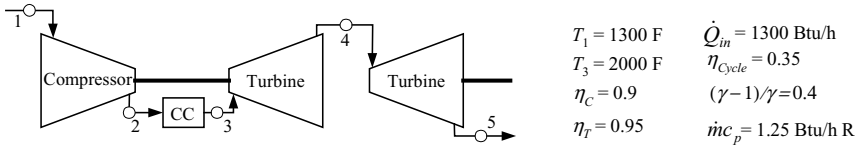
18. A gas turbine power plant consisting of high pressure (HP) and low pressure (LP) compressors and turbines is shown in the figure. The HP and LP turbines drive the HP and LP compressors, respectively. a) Place letters “a” through “j” on the accompanying T - s diagram and b) use the given data to find the net work and the cycle thermal efficiency. Data: $T_a = 20^\circ\text{C}$, $T_f = 850^\circ\text{C}$, $(r_p)_{C1} = (r_p)_{C2} = 4$, $\eta_c = 80\%$, $\eta_t = 85\%$, and the heat exchanger effectiveness is 75%.



19. In the Brayton cycle shown in the figure, pressure drop at the heat sink is negligible hence, $r_c \neq r_T$. a) Draw the T - s diagram and explicitly show the differences between the pressure levels of states 1 and 3. b) Use the given data to find r_c and r_T , assuming the working fluid is an ideal gas.

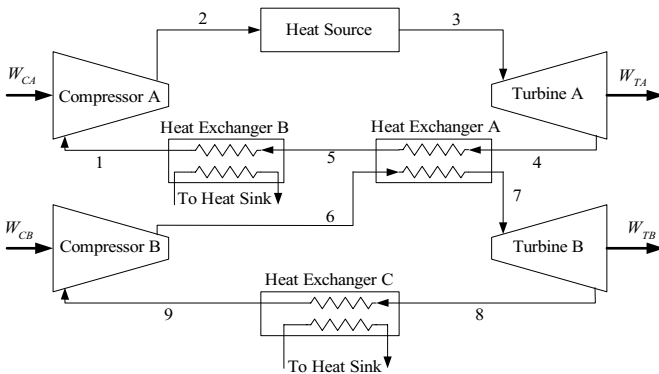


20. Air enters the compressor of a helicopter engine. The pressurized air is then delivered to the combustion chamber (CC). High energy mixtures then enter the first turbine, referred to as the *gas generator* and then the second turbine, referred to as the *power turbine*. Use the data given and find the power produced by the system in the following two cases: a) assume all the processes are ideal and b) assume the compressor and the turbine efficiencies are 90% and 85%, respectively.



21. Two ideal Brayton cycles are shown in the Figure. The working fluid is cooled in Heat Exchanger A prior to entering the heat sink (Heat Exchanger B). Heat Exchanger A is the heat source for a simple Brayton cycle. a) Draw the T - s diagram for this combined cycle, b) find the pressure ratio of turbine B, which maximizes the cycle thermal efficiency, and c) find the cycle thermal efficiency.

Data: $T_1 = T_9 = 5 \text{ C}$, $T_3 = 700 \text{ C}$, $T_4 - T_7 = 15 \text{ C}$, $P_2 = 4P_1$, $c_p = 5.23 \text{ kJ/kg}$, $\gamma = 1.658$. $\dot{m}_2 = 2\dot{m}_6$. The working fluid in both cycles is the same.



22. In a steam power plant, saturated steam at a pressure of 400 psia enters the turbine. The condenser pressure is 1 psia. Find the ideal Rankine cycle and the maximum thermal efficiency. [Ans: 33% and 38%]

23. Saturated steam in a Rankine cycle enters the turbine at 850 psia and leaves the condenser at 3.5 inches of Mercury (in Hg). Find: a) thermal efficiency of the cycle, b) power obtained from the turbine for steam mass flow rate of 11E6 lbm/hr. [Ans.: a) 34.8%, b) 1246 MW]

24. Superheated steam at a pressure of 4 MPa and a temperature of 350 C leaves the heat source and enters the turbine. The heat sink is at a pressure of 10 kPa. Calculate net work produced by the cycle and the cycle efficiency.

25. In Example IIb.2.1, we use an open FWH in conjunction with some extraction steam to heat up the feedwater. For the FWH operating at 250 psia, find the revised thermal efficiency and compare your calculated value with the results obtained in Example IIb.2.5. [Ans.: 0.385]

26. Saturated steam enters a turbine at 1000 psia. Heat is rejected in the condenser at 1 psia. Find thermal efficiency for a regenerative cycle, using a moisture separator and an open feedwater heater at 200 psia. Also find the steam extraction fraction, total pump work, and total turbine work. [Ans.: $\eta = 0.399$, $y = 0.2528$, $w_p = -3.098$ Btu/lbm, $w_t = 336.39$ Btu/lbm,]

27. In a Rankine cycle, dry saturated steam ($x = 100\%$), enters the turbine at 8 MPa and saturated liquid ($x = 0\%$) leaves the condenser at 0.008 MPa. Net power produced by this cycle is 100 MW. Find the turbine work, the cycle mass flow rate, the rate of heat transfer to the cycle, the rate of heat removal from the cycle, and thermal efficiency. [Ans.: $\eta_{th} = 37.1\%$ and mass flow rate = $3.77E5$ kg/h].

28. Perform a parametric study for a steam power plant operating between pressures of 7 MPa and 7 kPa on an ideal Rankine cycle. Use feedwater heater pressure as the variable. Produce plots similar to Figure I Ib.2.3 for such parameters as fraction of the steam extraction, pump work, and turbine work.

29. Consider the secondary side of a simplified PWR plant consisting only of the steam generator, turbine, condenser, and the feedwater pump. This plant is operating in an ideal Rankine cycle and producing dry saturated steam at a rate of $5.674E6$ kg/h and at a pressure of 7 MPa. A two-phase mixture leaves turbine and enters the condenser at 0.0075 MPa. The feedwater pump demands 9.4 kJ/kg at steady state to pump water from the condenser to the steam generator. A river flowing adjacent to the plant provides the cooling water to the condenser. According to regulations, the rise in the temperature of the river water exiting the plant must not exceed 8 C. Find a) plant thermal efficiency, b) maximum efficiency, and c) the flow rate of the circulating water through the condenser tubes. [Ans.: 36.7%, 43.9%, and $9.3E8$ kg/h].

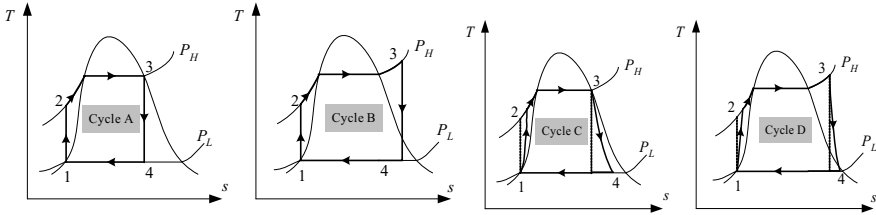
30. Superheated steam leaves the boiler of an ideal regenerative cycle at 600 psia and 800 F. Pressure in the feedwater heater and in the condenser is 60 psia and 1 psia, respectively. Find η_{th} . [Ans.: 39.1%].

31. Consider an air standard Otto cycle. The compression ratio is 8. At the beginning of the compression stroke, pressure is at 14.7 psia and 60 F. The heat transfer to the air per cycle is 800 Btu/lbm. Find the cycle thermal efficiency. [Ans.: $\eta_{th} = 56\%$].

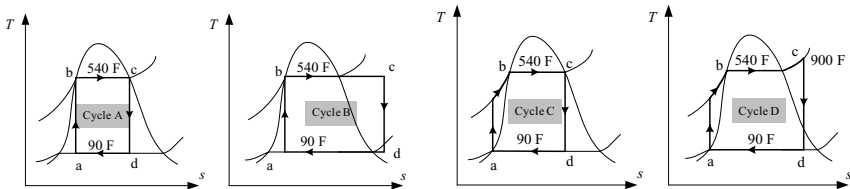
32. In a hypothetical 350 MWe nuclear power plant, cold water enters the reactor, steam leaves the reactor to enter the turbine, and the hot water from the turbine is returned to a nearby lake. Use the given data to find a) the temperature of the water at the inlet and outlet of the core, b) the maximum available work, c) the governing equation for the lake water temperature while ignoring any heat transfer by evaporation, and d) the plant lifetime based on the lake water temperature not exceeding 60 F.

Data: reactor power = 350 MWe, thermal efficiency = 0.333, temperature of water leaving the plant = 150 F, water mass flow rate = $1E4$ lbm/s, lake water volume = $1E12$ ft³, lowest water temperature = 50 F.

33. For the ideal Rankine cycles shown in the figure, find the minimum number of properties that we need to know in order to solve for the rest of unknowns such as pressures, temperatures, net work, and thermal efficiency. [Ans.: For cycle A we need, P_H and P_L . For Cycle D, we need P_H , P_L , T_3 , η_{turbine} , and η_{pump}].



34. Four designs for a steam power plant are shown in the figure. a) Find the heat supplied, the heat rejected, the net work, and the cycle thermodynamic efficiency for each design. b) Compare the results and comment on the advantage of each design.



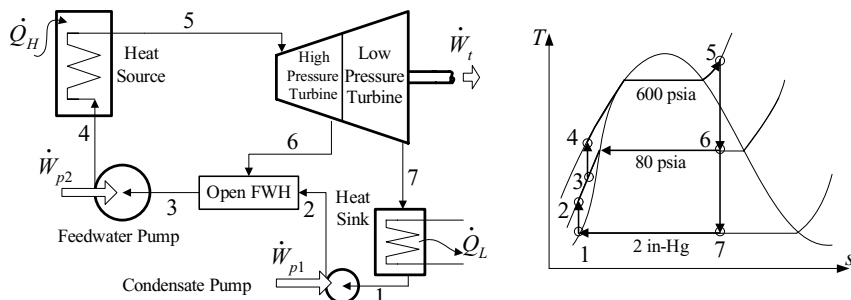
35. Find the efficiency of an ideal Rankine cycle using a steam temperature of 500 C and a condenser pressure of 0.1 bar. Try three steam pressures of 20 bar, 50 bar, and 100 bar. [Ans.: 34%, 38%, and 40%].

36. Find the efficiency of an ideal Rankine cycle in which superheated steam enters the turbine at 773 K and 40 bar. Try three condenser pressures of 2 bar, 0.5 bar, and 0.05 bar. [Ans.: 0.25, 0.31, and 0.38].

37. In a Rankine cycle, steam enters the turbine at 160 bar and 823 K. Pressure in the condenser is 0.05 bar. Find the cycle efficiency for the following isentropic efficiencies; $\eta_{\text{turbine}} = 0.88$, $\eta_{\text{pump}} = 0.9$.

38. In a regenerative modified Rankine cycle (Figure Iib.2.5), steam leaves the boiler and enters the turbine at 8 MPa and 753 K. Pressure in the open feedwater heater and the condenser are 0.7 MPa and 0.008 MPa, respectively. Water entering the feedwater pump is saturated. Each turbine has an isentropic efficiency of 0.85. a) Find the plant thermal efficiency, b) given a steam mass flow rate of 1E5 kg/h, find the net power.

39. A regenerative-modified Rankine cycle with moisture separator is shown in the figure. Use the given data in the figure to find the net cycle efficiency. Ignore the pump work.



40. Saturated steam at 1000 psia enters the high pressure turbine of an ideal regenerative Rankine cycle. The cycle is equipped with a moisture separator, delivering saturated water to an open feedwater heater at 100 psia. Saturated steam from the feedwater heater enters the low pressure turbine. Pressure in the condenser is 1 psia. The condensate leaving the open feedwater heater is saturated water. Draw the cycle schematic and the corresponding T - s diagram. Find a) the cycle thermal efficiency, b) the work per unit mass flow rate of water consumed by the condensate pump, c) the work per unit mass flow rate of water consumed by the feedwater pump, d) the work per unit mass flow rate of water produced by the high pressure turbine, e) the work per unit mass flow rate of water produced by the low pressure turbine, f) the heat per unit mass flow rate of water delivered to the heat source, and g) the fraction of steam used as the extracted steam in the open feedwater heater. [Ans.: 40.2%, 0.2 Btu/lbm, 3 Btu/lbm, 172.6 Btu/lbm, 188.6 Btu/lbm, 891.4 Btu/lbm, 20.5%].

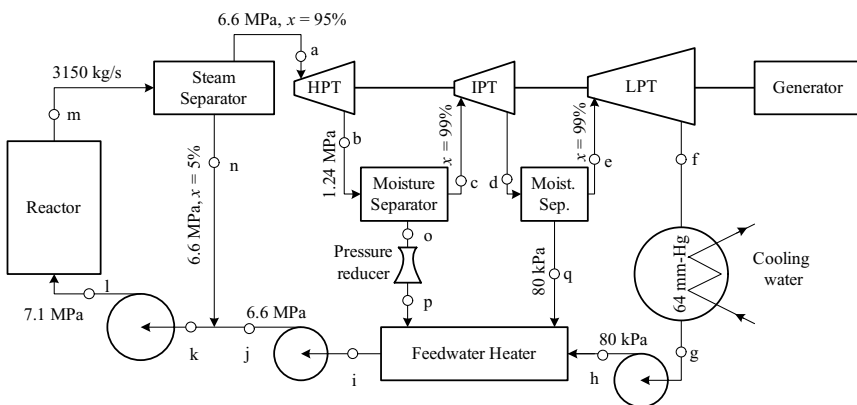
41. Superheated steam at 7.585 MPa (1100 psia) and 616.3 K (650 F) enters the high pressure turbine of an ideal regenerative Rankine cycle. The cycle is equipped with a moisture separator, delivering saturated water to an open feedwater heater at 0.689 MPa (100 psia). Saturated steam from the feedwater heater enters the low pressure turbine. Pressure in the condenser is 7 kPa (1 psia). The condensate leaving the open feedwater heater is saturated water. Find a) the cycle thermal efficiency, b) the work per unit mass flow rate of water consumed by the condensate pump, c) the work per unit mass flow rate of water consumed by the feedwater pump, d) the work per unit mass flow rate of water produced by the high pressure turbine, e) the work per unit mass flow rate of water produced by the low pressure turbine, f) the heat per unit mass flow rate of water delivered to the heat source, and g) the fraction of steam used as the extracted steam in the open feedwater heater. [Ans.: 41.1%, 0.465 kJ/kg, 7 kJ/kg, 469 kJ/kg, 474.7 kJ/kg, 2278.7 kJ/kg, 20.5%].

42. Saturated steam at 7.585 MPa (1100 psia) and 616.3 K (650 F) enters the high pressure turbine of an ideal regenerative Rankine cycle. The cycle is equipped

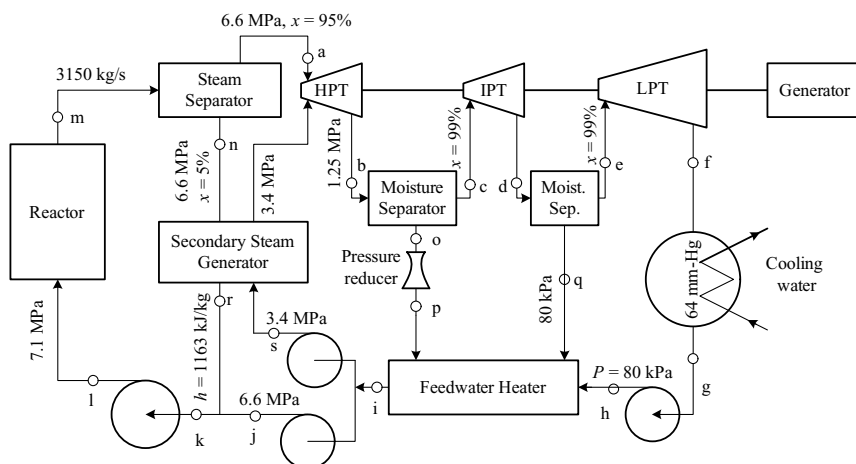
with a moisture separator, delivering saturated water to an open feedwater heater at 0.689 MPa (100 psia). Only 18% of the steam leaving the moisture separator is used in an open feedwater heater. Pressure in the condenser is 7 kPa (1 psia). Find a) the cycle thermal efficiency, b) the work per unit mass flow rate of water consumed by the condensate pump, c) the work per unit mass flow rate of water consumed by the feedwater pump, d) the work per unit mass flow rate of water produced by the high pressure turbine, e) the work per unit mass flow rate of water produced by the low pressure turbine, f) the heat per unit mass flow rate of water delivered to the heat source, and g) the enthalpy of water leaving the open feedwater heater. [Ans.: 39.3%, 0.465 kJ/kg, 62.8 kJ/kg, 469 kJ/kg, 489.4 kJ/kg, 2278.7 kJ/kg, 638.2 kJ/kg].

43. Solve problem 41 assuming an isentropic efficiency of 85% for each pump and 90% for each turbine. [Ans.: 35%, 0.697 kJ/kg, 63.96 kJ/kg, 422.15 kJ/kg, 440.3 kJ/kg, 2277.52 kJ/kg, 640.8 kJ/kg].

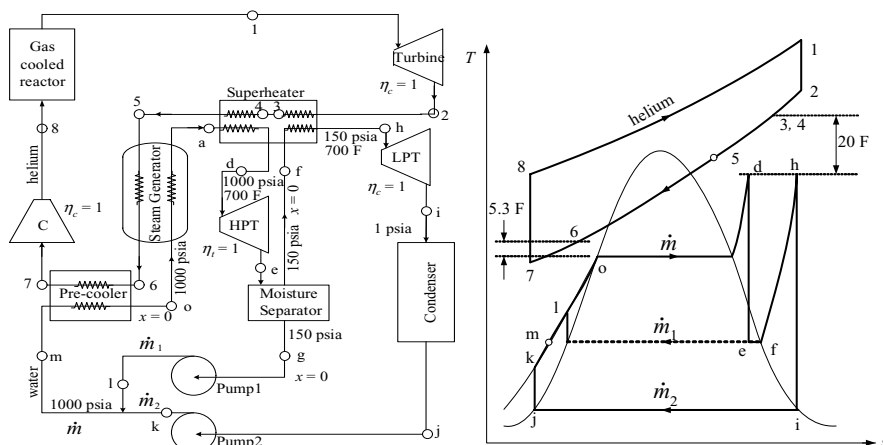
44. The schematic diagrams of two suggested designs for a boiling water reactor are shown in the figures. One uses a direct cycle and the other a dual cycle. The outlet conditions for both designs are the same. In both designs, steam leaves the steam separator assembly with a quality of 95% to enter the high pressure turbine (HPT) while the saturated liquid with 95 weight percent being recirculated to the reactor. The vapor is expanded successively in the HPT, the intermediate pressure turbine (IPT), and the low pressure turbine (LPT) before entering the condenser being at a 64 mm-Hg. Moisture separators between the HPT and IPT and between the IPT and LPT reduce the steam moisture to 1%; the separated liquid is used to heat the feedwater in an open feedwater heater. The heated condensate and the recirculated water from the steam separator are pumped through the reactor. Assume 100% efficiency in all pumps, and neglect pressure losses in the moisture separators. Take all turbine efficiencies as 75%.



In the dual cycle design, an additional steam generator is used. The 6.6 MPa saturated liquid from the steam separator produces saturated steam at 3.4 MPa in the secondary steam generator, its enthalpy being reduced to 1163 kJ/kg. The 3.4 MPa steam is introduced to the HPT at the appropriate stage with perfect mixing. Sketch the T - s diagram for the direct and dual cycles.



45. A combined Brayton – Rankine cycle is shown in the figure. Calculate a) all the flow rates shown in the diagram, b) all terms that are required to find the cycle thermal efficiency. Note the relative temperature relations; $T_6 = T_o + 5.3$ F, $T_4 = T_d + 20$ F.



IIC. Mixtures

Thermodynamic systems often include more than one component. For example, the combustion of fossil fuels results in a mixture product of several gases. Also the analyses of a PWR pressurizer and nuclear plant containment require consideration of such non-condensable gases as air in contact with water vapor. In this chapter we first study the fundamental relations related to mixtures and then apply these relations to the analysis of such interesting topics as conditioning a mixture of moist air, response of pressure suppression systems to pressurization, and the operation of cooling towers. We also study the pressure and temperature of a PWR containment following such events as the rupture of pipes carrying high energy fluids inside the containment.

Gas mixtures can be divided into two major categories: non-reactive and reactive gases. Moist air on a humid day is an example of non-reactive gases and a combustible mixture in the cylinder of an internal combustion engine is an example of the reactive gases. The non-reactive gases can be further divided into two categories: mixture of real gases and mixture of ideal gases. Air, for example, may be considered as a mixture of ideal gases. In this chapter we deal only with the mixture of non-reactive ideal gases.

1. Mixture of Non-reactive Ideal Gases

Dry air is a good example of a mixture of non-reactive ideal gases. The mole fraction of each component of dry air is shown in Table IIC.1.1.

Table IIC.1.1. Composition of dry air

Component	Mole Fraction (%)
Nitrogen	78.08
Oxygen	20.95
Argon	0.93
Carbon Dioxide	0.03
Neon, Helium, Methane, etc.	0.01

Due to the importance of air in industrial applications, air properties are identified and tabulated at various pressures and temperatures. However, in general, where various gases at various mole fractions may mix, we must find an easier way to represent the property of the mixture of gases. That is to say that we must use the properties of the pure substances that constitute the mixture and find equivalent properties as if the mixture itself is a pure substance. For example, consider a system containing N non-reactive ideal gases. There are two models to find the representative properties for this system: the *Dalton* and the *Amagat* models. Regardless of the model we use, the total number of moles in the system n is given as

$$n = \sum_{i=1}^N n_i \quad \text{IIC.1.1}$$

where n_i is the number of moles of component i and N is the number of components comprising the mixture.

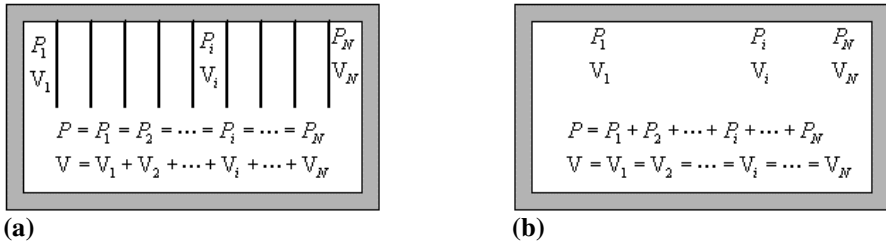


Figure IIC.1.1. (a) Amagat and (b) Dalton models for non-reactive mixture of ideal gases

Amagat Model, Equal Pressure and Temperature

Consider the system of gases shown in Figure IIC.1.1(a). In the Amagat model, all the non-reactive ideal gases are at the same pressure and temperature but at different volume so that the summation of all the volumes becomes equal to the volume of the system. This can be verified by applying the ideal gas law to each volume of gas:

$$n_i = \frac{PV_i}{R_u T}$$

and then substitute in Equation IIC.1.1, to get:

$$V = \sum_{i=1}^N V_i \quad \text{IIC.1.2}$$

See Problem 3 at the end of this section for the applicability of the Amagat model to non-ideal gases.

Dalton Model, Equal Volume and Temperature

Consider the system of gases shown in Figure IIC.1.1(b). In the Dalton model, all non-reactive ideal gases have the same volume and temperature but are at different pressure so that the summation of all pressures becomes equal to the pressure of the system. This can be verified by applying the ideal gas law to each gas:

$$n_i = \frac{P_i V}{R_u T}$$

and substitute in Equation IIc.1.1, to get:

$$P = \sum_{i=1}^N P_i \quad \text{IIc.1.3}$$

where V_i in Equations IIc.1.2, and P_i in Equation IIc.1.3 are referred to as partial volume and partial pressure, respectively. Similarly, V in Equations IIc.1.2, and P in Equation IIc.1.3 are referred to as total volume and total pressure, respectively. The Dalton model is more commonly applied to the mixture of ideal gases than the Amagat model. It is important to note that for both Amagat and Dalton models

$$T = T_1 = T_2 = \dots = T_i = \dots = T_N.$$

Example IIc.1.1. A tank having a volume of 10 m^3 is filled with nitrogen and 5 kg of carbon dioxide at a pressure and temperature of 140 kPa and 70 C, respectively. Find the partial volumes according to the Amagat model and the partial pressures according to the Dalton model.

Solution: Total volume and pressure are given. In both models, gases are at thermal equilibrium at 70 C.

a) *Amagat Model:* To find the partial volumes, we apply the ideal gas law to carbon dioxide:

$$V_{\text{CO}_2} = m \left(\frac{R_u}{M_{\text{CO}_2}} \right) \frac{T}{P} = 5 \left(\frac{8.31434}{44.01} \right) \frac{70 + 273}{140} = 2.31 \text{ m}^3$$

According to the Amagat model $V_{\text{N}_2} = V - V_{\text{CO}_2} = 10 - 2.31 = 7.69 \text{ m}^3$.

b) *Dalton Model:* We can use similar procedure to find the partial pressures from the Dalton model by applying the ideal gas law to carbon dioxide:

$$P_{\text{CO}_2} = m \left(\frac{R_u}{M_{\text{CO}_2}} \right) \frac{T}{P} = 5 \left(\frac{8.31434}{44.01} \right) \frac{70 + 273}{10} = 32.4 \text{ kPa}$$

Therefore, $P_{\text{N}_2} = P - P_{\text{CO}_2} = 140 - 32.4 = 107.6 \text{ kPa}$

Application of Dalton Model to Moist Air

The term moist air refers to a mixture of dry air, treated as a pure substance, and water vapor. Consider a volume containing moist air with n_a moles of dry air and n_v moles of water vapor at pressure P and temperature T . The total number of moles in this volume is found as

$$n = n_a + n_v$$

The mole fraction of air is given as:

$$y_a = n_a/n$$

and the mole fraction of water vapor as:

$$y_v = n_v/n$$

where subscripts a and v stand for air and water vapor, respectively. Total pressure of the moist air, partial pressure of the dry air, and partial pressure of water vapor are found as:

$$P = \frac{nR_u T}{V}, \quad P_a = \frac{n_a R_u T}{V}, \quad P_v = \frac{n_v R_u T}{V} \quad \text{IIC.1.4}$$

respectively. From these relations we conclude that $P_v = y_v P$ and $P_a = y_a P$.

Example IIC.1.2. A tank of volume 10 m^3 contains a mixture of air and superheated steam at a total pressure of 355 kPa and temperature of 100 C. The tank contains 0.05 lbmole of steam and 0.8 lbmole of air. Find the air and steam partial pressures.

Solution: Total number of moles of the mixture is $n = n_v + n_a = 0.05 + 0.8 = 0.85$. Therefore, the mole fractions of vapor and air are $y_v = 0.05/0.85 = 0.059$ and $y_a = 0.8/0.85 = 0.94$, respectively. This results in the vapor and air partial pressures of $P_v = y_v P = 21 \text{ kPa}$ and $P_a = y_a P = 334 \text{ kPa}$, respectively.

In the next example, we calculate the component masses of a mixture from partial pressures.

Example IIC.1.3. A large dry containment of a PWR has a volume of $2\text{E}6 \text{ ft}^3$. At normal operation, the mixture of air and superheated steam is at a total pressure of 14.7 psia and temperature of 120 F. If the partial pressure of superheated steam is 0.2 psia, find the masses of air and steam in the containment.

Solution: To find the masses, let's assume that both steam and air can be treated as ideal gases. Hence, for steam:

$$m_v = \frac{P_v V}{(R_u / M_v) T} = \frac{(0.2 \times 144.00) \times (2 \times 10^6)}{(1545/18)(120 + 460)} = 1157 \text{ lbm}$$

Since steam partial pressure is given, it implies that the calculation should be based on the Dalton model. Hence, $P_a = P - P_v = 14.7 - 0.2 = 14.5 \text{ psia}$. Therefore, for air:

$$m_a = \frac{P_a V}{(R_u / M_a) T} = \frac{(14.5 \times 144.00) \times (2 \times 10^6)}{(1545/28.97)(120 + 460)} = 135,000 \text{ lbm}$$

Calculation of Mixture Properties

To simplify dealing with mixtures, we calculate average mixture properties from the properties of the components comprising the mixture. The component properties are either based on mole fraction or mass fraction:

$$C_v = n\bar{c}_v = n \sum_i y_i \bar{c}_{v,i}(T, P_i) = mc_v = m \sum_i x_i c_{v,i}(T, P_i) \quad \text{IIc.1.5}$$

$$C_p = n\bar{c}_p = n \sum_i y_i \bar{c}_{p,i}(T, P_i) = mc_p = m \sum_i x_i c_{p,i}(T, P_i) \quad \text{IIc.1.6}$$

$$U = n\bar{u} = n \sum_i y_i \bar{u}_i(T, P_i) = mu = m \sum_i x_i u_i(T, P_i) \quad \text{IIc.1.7}$$

$$H = n\bar{h} = n \sum_i y_i \bar{h}_i(T, P_i) = mh = m \sum_i x_i h_i(T, P_i) \quad \text{IIc.1.8}$$

Here, the mass fraction of each component is defined as the ratio of the mass of that component to the total mass of the mixture. We examine the application of these relations in the following example.

Example IIc.1.4. Assuming air consists of only N₂, O₂, and Argon, find u , h , and c_p of air at 1 atm and $T = 80^\circ\text{F}$ (540°R) for the percentage specified below:

Component i	Molecular weight M	Volume fraction V_i/V	Mole fraction y_i	Mass fraction x_i
N ₂	28.013	0.7803	0.7803	0.7546
O ₂	31.999	0.2099	0.2099	0.2319
A	39.946	0.0098	0.0098	0.0135

Solution: We first calculate the mixture molecular weight:

$$M = \sum_i y_i M_i = 0.7803(28.013) + 0.2099(31.999) + 0.0098(39.948) = 28.967 \text{ lb/lbmole}$$

Using M , we calculate x_i according to:

$$x_i = y_i M_i / M$$

The results are listed in the above table. Obtaining c_v and c_p from Table A.II.5 (BU), for the mixture specific internal energy, we find:

$$u = \sum_i x_i u_i = 0.7546(540 \times 0.1774) + 0.2319(540 \times 0.157) + 0.0135(540 \times 0.0746) =$$

$$92.5 \text{ Btu/lbm}$$

for specific enthalpy we find:

$$h = \sum_i x_i h_i = 0.7546(540 \times 0.2483) + 0.2319(540 \times 0.2191) + 0.0135(540 \times 0.1244) =$$

$$129.5 \text{ Btu/lbm}$$

and for specific heat we find:

$$c_p = \sum_i x_i c_{pi} = 0.7546(0.2483) + 0.2319(0.2191) + 0.0135(0.1244) = 0.24 \text{ Btu/(lbm R)}$$

Example Ilc.1.5. A cylinder contains 1 lbm of CO₂ and 2 lbm of N₂ at 20 psia and 100 F. In a polytropic process ($n_{poly} = 1.3$), the content is compressed to 60 psia. Find the value of work and heat transfer.

Solution. The work done on the system can be found from Equation IIa.4.4;

$$W = \frac{P_2 V_2 - P_1 V_1}{1 - n_{poly}} = \frac{m(R_u / M)(T_2 - T_1)}{1 - n_{poly}}$$

Where m and M are the mixture mass and molecular weight. We need to find m , M , and T_2 . The mixture mass is found from:

$$m = m_{\text{CO}_2} + m_{\text{N}_2} = 1 + 2 = 3 \text{ lbm}$$

To find the molecular weight, we must first find total number of molecules:

$$n_{\text{CO}_2} = \frac{m_{\text{CO}_2}}{M_{\text{CO}_2}} = \frac{1}{44} = 0.023 \text{ lbmol}$$

$$n_{\text{N}_2} = \frac{m_{\text{N}_2}}{M_{\text{N}_2}} = \frac{2}{28} = 0.071 \text{ lbmol}$$

Therefore, $N = 0.023 + 0.071 = 0.094 \text{ lbmol}$ and $M = m/N = 3/0.094 = 31.91$. Mixture temperature following compression is found from:

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{(n_{poly}-1)/n_{poly}} = 560(3)^{(1.3-1)/1.3} = 722 \text{ R}$$

Substituting, we find the amount of work delivered to the system as:

$$W = \frac{m(R_u / M)(T_2 - T_1)}{1 - n_{poly}} = \frac{3(1545 / 31.91)(722 - 560)}{1 - 1.3} = -78436 \text{ ft lbf} = -100.8 \text{ Btu}$$

where the minus sign confirms that work is delivered to the system. The heat transfer is found from the first law of thermodynamics:

$$Q = W + \Delta U$$

We calculate ΔU from:

$$\Delta U = \left(\sum_{i=1}^N m_i c_{vi} \right) (T_2 - T_1) = (1 \times 0.158 + 2 \times 0.177)(722 - 660) = 31.74 \text{ Btu}$$

Therefore, the amount of heat transferred to the surroundings is found as:

$$Q = -100.8 + 31.74 = -69 \text{ Btu.}$$

Example IIc.1.6. A gas tank contains a mixture of 1.35 kmol CO_2 and 4.8 kmol of air at 1.2 bar and 37 C. Assuming air by volume consists of 21% O_2 and 79% N_2 , find:

- the masses of N_2 , O_2 , and CO_2 as well as the total mass
- the percentage of carbon in the mixture by mass
- the molecular weight of the mixture
- specific volume of the mixture

Solution. a) We first find the number of moles:

For $n_{\text{CO}_2} = 1.35$ kmol, $n_{\text{O}_2} = 4.8 \times 0.21 = 0.97$ kmol, and $n_{\text{N}_2} = 4.8 \times 0.79 = 3.79$ kmol. Having number of moles, we then find the masses from $m = nM$. Hence, for nitrogen $m_{\text{N}_2} = 3.79 \times 28 = 106.2$ kg, for oxygen

Mass of mixture: $m = m_{\text{N}_2} + m_{\text{O}_2} + m_{\text{CO}_2} = 106.2 + 31 + 59.4 = 196.6$ kg.

b) $m_C = [(12/44) \times 59.4]/196.6 = 8\%$

c) To find M , we need to find total number of moles and the mole fraction of each component.

$$n = n_{\text{CO}_2} + n_{\text{O}_2} + n_{\text{N}_2} = 3.97 + 0.97 + 1.35 = 6.29 \text{ kmol}$$

$$y_{\text{N}_2} = 3.97/6.29 = 0.63$$

$$y_{\text{O}_2} = 0.97/6.29 = 0.16$$

$$y_{\text{CO}_2} = 1.35/6.29 = 0.21$$

$$M = 0.63(28) + 0.16(32) + 0.21(44) = 34.82 \text{ kg/kmol}$$

$$\text{d) } v = RT/P = (R_u/M)T/P = (8314.5/34.82) \times (273 + 37)/(1.2 \times 10^5) = 0.62 \text{ m}^3/\text{kg}.$$

2. Gases in Contact with Ice, Water, and Steam

Moist air is one of the most important mixtures for industrial applications. Let's consider a general case of a system consisting of non-condensable gases in contact with ice, water, and water vapor, as shown in Figure IIc.2.1. The system therefore consists of three regions. The water region is generally referred to as the *pool*. The gas region consists of gases, vapor, and water droplets. Gases may include any combination of air and such other gases as carbon monoxide, ammonia, ethanol, etc. Depending on the process, which such system may undergo, various phases in this system would interact. For example, the superheated steam may condense on the droplets and droplets may vaporize in contact with hot gases. Also water may evaporate at the interface, steam would condense on the ice surface, and ice would melt in contact with warmer water and gases. Having defined this general case, in the following sections, we deal with specific cases of a mixture of air and water vapor as well as the mixture of moist air being in contact with

a pool of water. Therefore, we exclude the presence of the ice region. Additionally, if there is a pool region, we assume no gas is dissolved in the pool.

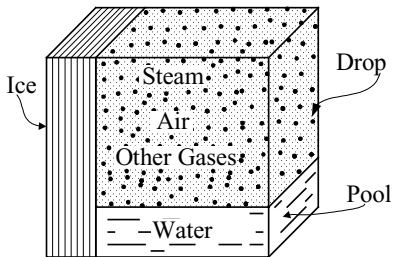


Figure IIc.2.1. Generalization of a thermodynamic system containing water and gas

Relative Humidity, a Measure of Moisture Content

Let's limit the discussion to the control volume representing the gas region of Figure IIc.2.1. We further limit the discussion to a case when the gas region consists only of a mixture of air and water vapor. This moist air mixture has n_a moles of dry air and n_v moles of water vapor at pressure P and temperature T .

<div style="display: flex; justify-content: space-between;"> Water vapor Air </div> <div style="text-align: center; padding: 5px;"> Moist Air </div> <div style="display: flex; justify-content: space-between; padding: 5px;"> $n = n_a + n_v$ $P = P_a + P_v$ </div>

Let's now bring the water vapor to saturation while maintaining the temperature and total pressure of the mixture at the above values. For the mixture of moist air, the relative humidity is defined as:

$$\phi = \left(\frac{y_v}{y_g} \right)_{P, T}$$

where the saturation state is shown by subscript g and the mole fraction of saturated steam in the mixture by $y_g = n_g/n$. Since $P_v = y_v P$ and $P_g = y_g P$, relative humidity can be written as:

$$\phi = \frac{P_v}{P_g(T)} \tag{IIc.2.1}$$

Equation IIc.2.1 is shown in Figure IIc.2.2(a). In Figure IIc.2.2(b), a relative humidity of unity is obtained by adding steam and replacing some air to maintain the same total pressure as in Figure IIc.2.2(a).

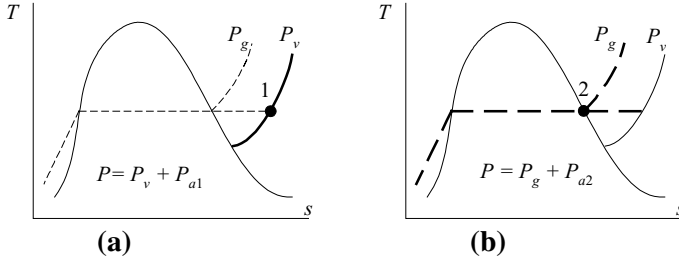


Figure IIc.2.2. (a) State of vapor in moist air and in (b) saturated mixture

Example IIc.2.1. A large dry containment of a PWR has a volume of $2\text{E}6 \text{ ft}^3$. At normal operation, the mixture of air and superheated steam is at a total pressure of 14.7 psia, temperature of 120 F, and relative humidity of 65%. Find the masses of air and steam in the containment.

Solution: To find the masses of air and steam, we need the partial pressure of each component. To find the partial pressure of steam, we use the relative humidity. $P_v = \phi P_g$. From the steam tables we find:

P_g (120 F) = 1.6927 psia. Therefore, $P_v = 0.65 (1.6927) = 1.1$ psia and $P_a = 14.7 - 1.1 = 13.6$ psia. Finally:

$$m_v = \frac{P_v V}{(R_u / M_v) T} = \frac{(1.1 \times 144.00) \times (2 \times 10^6)}{(1545 / 18)(120 + 460)} = 6364 \text{ lbm}$$

$$m_a = \frac{P_a V}{(R_u / M_a) T} = \frac{(13.6 \times 144.00) \times (2 \times 10^6)}{(1545 / 28.97)(120 + 460)} = 126,626 \text{ lbm}$$

Humidity Ratio or Specific Humidity

Another means of measuring the moisture content in moist air is calculating the humidity ratio, defined as the mass of the water vapor to the mass of dry air:

$$\omega = \frac{m_v}{m_a} = \frac{P_v V / (R_u / M_v) T}{P_a V / (R_u / M_a) T} = \frac{M_v P_v}{M_a P_a} = 0.622 \frac{P_v}{P - P_v} \quad \text{IIc.2.2}$$

Example IIc.2.2. Find the relative humidity for a sample of moist air at 14.7 psia and 80 F if the humidity ratio is 0.02.

Solution: From humidity ratio, we find $P_v = P / [1 + (0.622 / \omega)]$. Substituting for total pressure and for the humidity ratio, $P_v = 14.7 / (1 + 0.622 / 0.02) = 14.7 / 32.1 = 0.458$ psia. Also P_g (80 F) = 0.507 psia. Therefore, $\phi = 0.458 / 0.507 = 90\%$.

3. Processes Involving Moist Air

In this section we discuss isochoric, isobaric, and adiabatic processes involving moist air. We start with the isobaric process. Cooling down of moist air in many air-conditioning systems can be considered cooldown at constant pressure. The following example deals with calculating the rate of condensate produced in such systems.

Mixture Cooldown at Constant Pressure, Dew Point Temperature

To describe the dew point temperature, we consider unsaturated moist air at temperature T_1 . Steam in this mixture is superheated at state 1 (partial pressure P_1 and temperature T_1 in Figure IIC.3.1). Hence, the relative humidity is less than unity. State 2 shows saturated steam corresponding to temperature T_1 . If the moist air was at state 2, the mixture would have been saturated. The dew point of the mixture at state 1 is the temperature to which the mixture should be cooled down at constant pressure to become saturated. As shown in Figure IIC.3.1, temperature T_3 is the dew point temperature for the mixture at state 1, $T_3 = T_g(P_1)$. If any of the steam condenses, then saturated water appears at state 4. Further cooldown of the mixture occurs on the saturation line (State 5). Such cooldown results in lower steam partial pressure (P_{gs}) due to the appearance of condensate dropping out of the mixture (State 6).

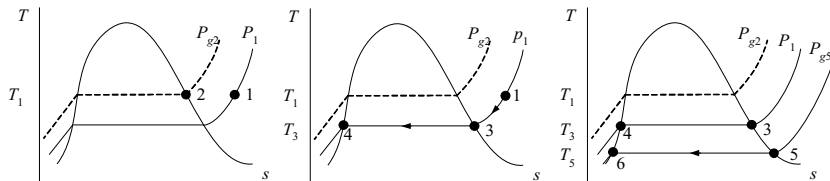


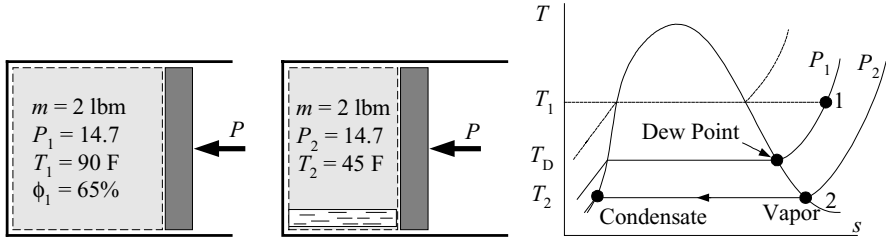
Figure IIC.3.1. Cooldown of unsaturated mixture to saturation

Example IIC.3.1. A large dry containment of a PWR has a free volume of 57000 m³. Following an event, the moist air in this containment reaches 1.5 atm, 130 C, and a relative humidity of 15%. Find the dew point temperature corresponding to this state.

Solution: To find the dew point temperature, we need to find the saturation temperature corresponding to the mixture partial pressure of steam (i.e., $T_{\text{Dew Point}} = T_g(P_v)$). To find P_v , we find $P_g(T_v) = P_g(130 \text{ C}) = 2.701 \text{ bar}$. So that:
 $P_v = 0.15(2.701) = 0.4 \text{ bar}$. The corresponding saturation temperature is:
 $T_g(0.4 \text{ bar}) = 75.8 \text{ C}$.

Example IIc.3.2. A 2-lbm sample of moist air is initially (state 1) at $P_1 = 14.7$ psia, $T_1 = 90$ F, and $\phi_1 = 65\%$. This mixture is cooled at constant pressure to $T_2 = 45$ F (state 2). Find a) the humidity ratio at state 1, b) the dew point temperature at states 1 and 2, c) the amount of condensate at state 2.

Solution: a) To find the initial humidity ratio we need to have P_v . This is found from the initial relative humidity. We first find $P_g(90 \text{ F}) = 0.698$ psia. Hence, $P_v = 0.65(0.698) = 0.4537$ psia, and $\omega_1 = 0.622 \times 0.4537 / (14.7 - 0.4537) = 0.02$.



b) The dew point temperature corresponding to state 1 is $T_{D1} = T_g(P_{v1}) \approx 76$ F and for state 2 is 45 F.

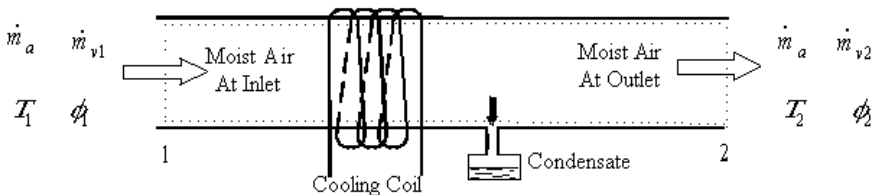
c) The mixture becomes saturated at $T_{D1} = 76$ F. Further decrease in temperature results in steam condensation. At state 2, $P_v = P_2 = P_g(T_2) = P_g(45 \text{ F}) = 0.14744$ psia. Since total pressure is kept constant,

$\omega_2 = 0.622 \times 0.14744 / (14.7 - 0.14744) = 0.0063$ hence, $m_{v2} = 0.0063 m_a$. We must find m_a . On one hand
 $m_a + m_{v1} = 2 \text{ lbm}$

On the other hand $m_{v1}/m_a = 0.02$. Solving this set we find, $m_a = 1.9608$ lbm and $m_{v1} = 0.0392$ lbm. Therefore, $m_{v2} = 0.0063 \times 1.9608 = 0.01235$ lbm. Hence, the mass of steam condensed in this process is:

$m_c = m_{v1} - m_{v2} = 0.0392 - 0.01235 = 0.02685$ lbm.

Example IIc.3.3. Moist air at 1 atm, 20 C, and a relative humidity of 70% enters a cooling duct at a rate of $1.3 \text{ m}^3/\text{s}$. Temperature of the saturated mixture at the exit of the cooling coil is 5 C. Assuming negligible pressure drop, find the mass flow rate of the condensate produced in the cooling duct.



Solution: The condensate mass flow rate is calculated as $\dot{m}_c = \dot{m}_{v1} - \dot{m}_{v2}$. To find the vapor mass flow rates, we need to calculate the air mass flow rate and

then use Equation IIc.2.2. To calculate the air mass flow rate, we need air pressure,

$$P_{a1} = P - P_{v1} = P - \phi_1 P_g(T_1) = 1.01325 - 0.7(0.02339) = 0.997 \text{ bar.}$$

Hence, air mass flow rate is obtained from:

$$\dot{m}_a = \rho_a \dot{V}_a = \frac{P_a}{(R_u / M_a) T} \dot{V}_a$$

Substituting:

$$\dot{m}_a = \frac{0.997}{(0.08314 / 28.97)(273 + 20)} (1.3) = 1.54 \text{ kg/s}$$

To find the humidity ratios, we find vapor partial pressures at the inlet and outlet.

At the inlet:

$$P_{v1} = 0.7(0.02339) = 0.0164 \text{ bar}$$

and at the outlet the mixture is saturated

$$P_{v2} = 1.0(0.00872) = 0.00872 \text{ bar.}$$

$$\text{Therefore, } \omega_1 = 0.622(0.0164) / (1.01325 - 0.0164) = 0.01$$

$$\omega_2 = 0.622(0.0087) / (1.01325 - 0.0087) = 0.005399$$

Thus $\dot{m}_C = \dot{m}_{v1} - \dot{m}_{v2} = \dot{m}_a(\omega_1 - \omega_2)$ substituting for \dot{m}_a , the mass flow rate of condensate is found as:

$$\dot{m}_C = \dot{m}_a(\omega_1 - \omega_2) = 1.54(0.01 - 0.0087) = 0.00785 \text{ kg/s} = 25.5 \text{ kg/h.}$$

Mixture Cooldown at Constant Volume

We often encounter mixture cooldown at isochoric instead of isobaric process. This occurs when a non-deformable (rigid) volume contains a fixed amount of a mixture (state 1 on Figure IIc.3.2) and the volume is then subjected to cooldown. In this case, the temperature at which condensate appears (state 2 on Figure IIc.3.2) differs from the dew point temperature (State D on Figure IIc.3.2). To find the temperature corresponding to state 2, we note that at the moment that vapor becomes saturated at constant volume we have $v_{g2} = v_{v1}$. We wrote this relation based on the fact that both volume and all masses remain constant throughout the cooldown process. Since we know P_1 and T_1 we can find v_{v1} . Then from the steam tables, we can find the temperature corresponding to the saturated steam specific volume v_{g2} .

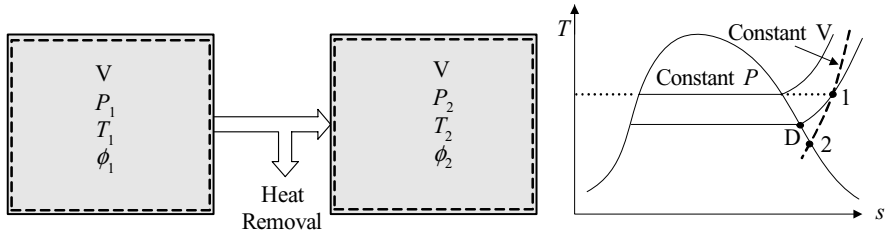
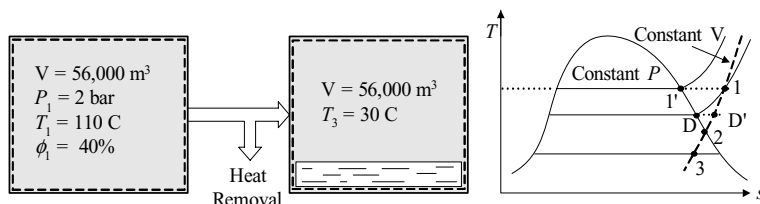


Figure IIc.3.2. Cooldown of moist air at constant volume

Example IIc.3.4. Moist air is contained in a volume of $56,000 \text{ m}^3$ at 2 bar, 110°C , and a relative humidity of 40%. This mixture is now cooled to 30°C . Find a) the dew point temperature, b) temperature at which vapor begins to condense, c) the amount of water condensed, and d) final pressure. States are shown in the figure.



Solution: a) We find the dew point temperature from $T_D = T_g(P_{v1})$. To find P_{v1} , we use the relative humidity. $P_{v1} = P_g(T_1) = P_g(110^\circ\text{C}) = 0.14327 \text{ MPa} = 1.4327 \text{ bar}$.

Hence, $P_{v1} = 0.40(1.4327) = 0.573 \text{ bar}$. We find $T_D = T_{\text{sat}}(P_{v1} = 0.573 \text{ bar}) = 84.38^\circ\text{C}$. Also note that $P_{a1} = 2 - 0.573 = 1.427 \text{ bar}$.

b) Since the cooldown is at constant volume, we know that the condensate first appears at T_2 because on the constant volume line, the pressure corresponding to T_D is smaller than the saturation pressure corresponding to the dew point temperature (i.e., $P_{D'} < P_D = P_g(T_D)$). Hence, vapor is superheated at $T_{D'} = T_D$ and $P_{D'}$. Temperature at which vapor begins to condense is found from $v_{g2} = v_1$, where v_1 is given by $P_1 v_1 = (R_u/M_a)T_1$. Subsequently, we find $v_1 = (0.08314/18)(110 + 273)/0.573 = 3.8 \text{ m}^3/\text{kg}$. This corresponds to a saturation pressure of $T_2 = 82.2^\circ\text{C}$, which is 2°C less than the dew point temperature.

c) To find the mass of the condensate, we again use the fact that cooldown is at a constant volume: $v_3 = v_2 = v_1$. From the steam tables, we find $v_f(30^\circ\text{C}) = 0.001004 \text{ m}^3/\text{kg}$ and $v_g(30^\circ\text{C}) = 32.89 \text{ m}^3/\text{kg}$. Steam quality at point 3 is found as $x_3 = (v - v_f)/v_{fg} = (3.8 - 0.001004)/(32.89 - 0.001004)$

d) The moist air volume at state 3 is $V_3 = 56,000 - 1702(0.001004) = 55998.3 \text{ m}^3$. $P_{\text{final}} = P_3 + P_{a3}$. Where $P_{a3} = m_a(R_u/M_a)T_3/V_3$. However, the dry air mass is found from $m_a = P_{a1}V_1/(R_u/M_a)T_1 = (2 - 0.573)(56,000) / (0.08314/28.97)(30 + 273) = 91898.4 \text{ kg}$. Therefore, $P_{a3} = 91898.4 (0.08314/28.97)(30 + 273)/55998.3 = 1.427 \text{ bar}$. Hence, $P_{\text{final}} = 1.427 + P_g(30^\circ\text{C}) = 1.427 + 0.0425 = 1.47 \text{ bar}$.

Humidification

In the analysis of moist air in closed systems undergoing constant pressure or constant volume processes we were able to determine conditions of the final state of the mixture from the equation of state. To find more information about the process, for example the amount of heat transfer in a constant volume process we would have to use the conservation equation of energy in addition to the equation of state. In general, we need to use the conservation equation of mass, conservation equation of energy, and the equation of state as applied to a control volume to study the thermal-hydraulic characteristics of air conditioning systems. The applicable equations for conservation of mass and energy are IIa.5.1 and IIa.6.5, respectively. For example, let's analyze heating and humidification of moist air as shown in Figure IIc.3.3.

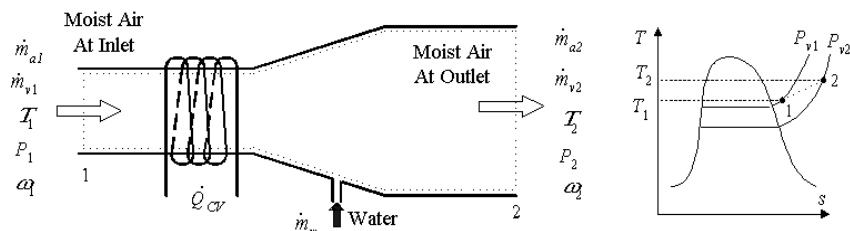


Figure Ilc.3.3. Control volume for conditioning a mixture of moist air

Considering steady state operation, the conservation equation of mass for dry air becomes:

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a \quad \text{Ilc.3.1}$$

and for water:

$$\dot{m}_{v1} + \dot{m}_w = \dot{m}_{v2} \quad \text{Ilc.3.2}$$

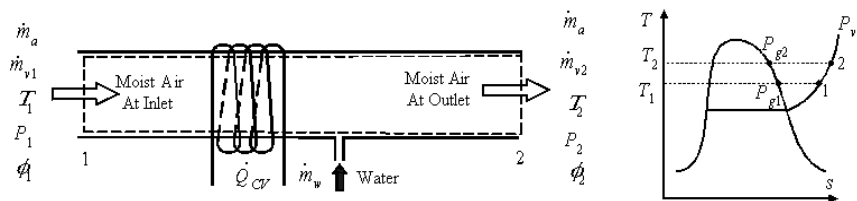
We apply the conservation equation of energy at steady state to the mixture to obtain:

$$(\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1}) + \dot{m}_w h_w + \dot{Q}_{CV} = (\dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}) \quad \text{Ilc.3.3}$$

where we assumed no net work and ignored the kinetic and potential energies. To simplify, we substitute for the vapor mass flow rate from $\dot{m}_v = \omega \dot{m}_a$ to obtain $\dot{m}_w = (\omega_2 - \omega_1)\dot{m}_a$. Substituting in Equation Ilc.3.3, we get:

$$\dot{Q}_{CV} / \dot{m}_a = c_{pa}(T_2 - T_1) + (\omega_2 h_{v2} - \omega_1 h_{v1}) - (\omega_2 - \omega_1)h_w \quad \text{Ilc.3.4}$$

Example Ilc.3.5. Moist air enters a heated duct at 15 psia, 50 F, 60% relative humidity and a volumetric flow rate of 5000 CFM. Water is sprayed into the moist air stream at a temperature of 80 F and a flow rate of 0.3 GPM. Assuming negligible pressure drop in the short duct, find the relative humidity at the outlet of the duct and the rate of heat transfer for steady state operation at $T_2 = 70$ F.



Solution: First, we find air density at the inlet to calculate the air mass flow rate, $\rho_{a1} = P_1 / (R_u / M_a) T_1 = 15(144) / [(1545/28.97)(50 + 460)] = 0.08 \text{ lbm/ft}^3$.

Hence, $\dot{m}_a = 0.08(5000)/60 = 6.62 \text{ lbm/s}$. We now calculate the inlet humidity

ratio $P_{v1} = 0.6P_g(50\text{ F}) = 0.6(0.178) = 0.11$ psia so that:

$$\omega_1 = 0.622(0.11)/(15 - 0.11) = 0.0045.$$

Mass flow rate of the injected water is;

$$\dot{m}_w = \rho_w \dot{V}_w = 62.2[0.3/(60 \times 7.481)] = 0.042 \text{ lbm/s} \quad (0.02 \text{ kg/s})$$

where the water density at 80 F is 62.2 lbm/ft³ and 7.481 is the conversion factor for ft³ to gallon. We find the humidity ratio at the outlet from:

$$\omega_2 = \omega_1 + (\dot{m}_w / \dot{m}_a) = 0.0045 + (0.042/6.62) = 0.$$

and $P_g(70\text{ F}) = 0.363$ psia (2.5 kPa) so that:

$$\phi_2 = 0.26/0.363 = 70\%.$$

Other parameters needed for Equation IIc.3.4 are water and vapor enthalpies. These can be found from the Steam Tables as $h_w = 48$ Btu/lbm, $h_{v1} = (P = 0.11, T = 50) = 1085$ Btu/lbm, and $h_{v2} = (0.11, 70) = 1092$ Btu/lbm (2540 kJ/kg). Substituting in Equation IIc.3.4, we get:

$$\dot{Q}_{CV} = 6.62[0.24(70 - 50) + (0.01 \times 1092 - 0.0045 \times 1085) - (0.01 - 0.0045)48] = 70 \text{ Btu/s} \quad (74 \text{ kW})$$

The Adiabatic Saturation Process

Another example of gases in contact with phases of water is the adiabatic saturation process. As shown in Figure IIc.3.4, moist air with an unknown relative humidity is passed over a pool of water contained in a well insulated duct. The mixture pressure and temperature at the inlet are specified. If the entering air is not saturated, some of the water in the pool would evaporate and enter into the mixture stream. For sufficiently long duct, the mixture at the outlet would be saturated. Temperature of the mixture at the outlet is less than the inlet temperature ($T_2 < T_1$) due to the fact that some energy of the mixture is used to evaporate water in the pool. This temperature is referred to as the *adiabatic saturation temperature* since the saturation of the mixture occurred without any need for heat transfer from the surroundings. Saturated make up water is added to the pool to maintain the process at steady state condition.

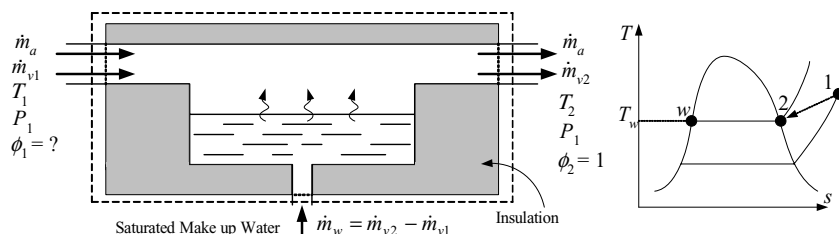


Figure IIc.3.4. Steady flow of moist air over a pool of water to produce saturated mixture

Measurement of Relative Humidity

We can use the above adiabatic saturation process to determine the unknown humidity ratio as well as the relative humidity. If the pressure drop in the duct is negligible, we can apply the same procedure that led to the derivation of Equation IIC.3.4 except for the heat transfer term which should be dropped:

$$c_{pa}(T_2 - T_1) + (\omega_2 h_{v2} - \omega_1 h_{v1}) - (\omega_2 - \omega_1)h_w = 0$$

We solve this equation for the unknown humidity ratio to obtain:

$$\omega_1 = \frac{c_{pa}(T_2 - T_1) + \omega_2(h_{v2} - h_f)}{(h_{v1} - h_f)} \quad \text{IIC.3.5}$$

Where ω_2 is given by Equation IIC.2.2. Note that $h_{v2} = h_g(T_2)$. Having ω_1 , we can find the unknown relative humidity from:

$$\phi = \frac{\omega}{(0.622 + \omega)} \frac{P}{P_g(T_v)} \quad \text{IIC.3.6}$$

Wet- and Dry-Bulb Temperatures

We measure the dry-bulb temperature of a mixture by a thermometer. To measure the wet-bulb temperature, we cover the bulb of the thermometer by a wet wick. We can then measure the wet-bulb temperature by either drawing the flow of the mixture over the wet bulb by a fan or moving the thermometer in the mixture. If the mixture is not saturated, some heat transfer takes place, transferring energy from the mixture to the wick for liquid evaporation. This results in the temperature shown by the thermometer to be lower than the dry-bulb temperature. We can then measure the humidity ratio by using Equations IIC.3.5, from which we can find the relative humidity.

Example IIC.3.6. Temperature of a room is measured as 72 F. The wet-bulb temperature is measured as 65 F. Find the relative humidity.

Solution: We can find the relative humidity from Equation IIC.1.20. This, in turn, requires the humidity ratio, which we can find from Equation IIC.1.19. Note that there is no make-up water hence Equation IIC.3.5 becomes

$$\omega_1 = [c_{pa}(T_2 - T_1) + \omega_2 h_g(T_2)] / h_{v1}(T_1)$$

To find ω_2 , we use Equation IIC.2.2: $\omega_2 = 0.622P_{v2} / (P - P_{v2})$ where $P = 14.7$ psia and $P_{v2} = P_g(T_2)$. For $T_2 = 65\text{F}$, $P_g(65) = 0.30545$ psia. Therefore, $\omega_2 = 0.622 (0.30545) / (14.77 - 0.30545) = 0.0132$. Then $\omega_1 = [0.24 (65 - 72) + 0.0132 (1089.9)] / 1093 = 0.0116$. Using in Equation IIC.3.6, we get:

$$\phi_1 = \frac{\omega_1}{(0.622 + \omega_1)} \frac{P}{P_g(T_{v1})} = \frac{0.0116}{(0.622 + 0.0116)} \frac{14.7}{0.38844} = 69\%$$

Note that we assumed $h_{v1} \cong h_g(T_1)$ to avoid iteration.

4. Charging and Discharging Rigid Volumes

This is a more general case of the topic discussed in Section 8 of Chapter IIa. The rigid container (i.e. constant volume process) initially contains moist air at specified pressure, temperature, and relative humidity. Fluid at a specified rate is now injected into the container. The intention is to find the equilibrium pressure and temperature. Similarly, we can consider a case where a valve is opened to allow a specified amount of the mixture to leave the container. Such a process, where the final equilibrium-state is not known, frequently occurs in common practice. Determination of the final equilibrium-state generally requires iteration with the steam tables. A special case is shown in Example IIc.4.2 where a mixture of water and steam enters a control volume and final pressure is sought.

Rigid Volumes Initially at Non-equilibrium Condition

First we consider a simple case where moist air is in contact with water. Note that both air and water are at the same temperature. Figure IIc.4.1(a) shows a system containing moist air with relative humidity less than 1. In such a system, water evaporates until the mixture of air and water vapor becomes saturated in steam and the system reaches equilibrium, Figure IIc.4.1(b). In such an equilibrium condition, all components are again at the same temperature albeit $T_2 < T_1$. In the thermodynamic analysis of such systems, we may assume that no gas is dissolved in water.

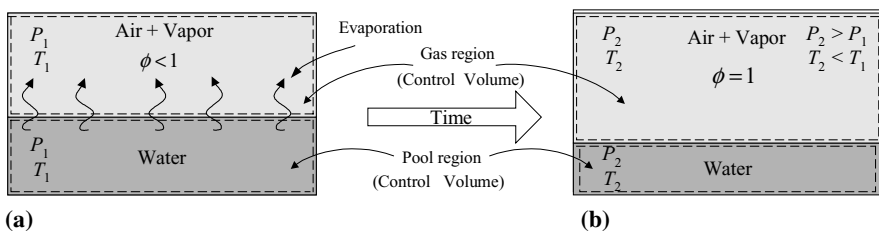
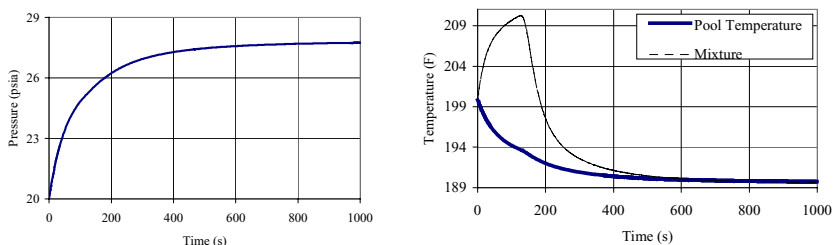


Figure IIc.4.1. (a) Water evaporation to reach and (b) Equilibrium state

Example IIc.4.1. The system in Figure IIc.4.1(a) has two distinct regions, the pool and the gas region. Subcooled water in the pool region is initially at pressure P_1 and temperature T_1 where $T_1 < T_{g1}(P_1)$. The mixture in the gas region is initially at total pressure P_1 , temperature T_1 , and relative humidity $\phi_1 < 1$. A thermally conducting plate separates these two regions. At time zero, the plate is removed

and the regions begin to exchange mass and energy. Assuming no heat transfer between the system and the surroundings, discuss the response of the system to the removal of the plate, Figure IIc.4.1(b).

Solution: What drives this transient is the gas region not being saturated. The transient begins at time zero, when the plate is removed and the regions are allowed to exchange mass and energy. To bring the gas region to saturation, water vaporizes, carrying saturated water enthalpy, $h_g(P)$ into the gas region. This increases pressure and temperature. Since, the energy for vaporization is provided by the pool water, this also causes water temperature and water level to drop. Water in the pool is subcooled at total pressure and will remain subcooled throughout the transient due to increasing pressure. However, at equilibrium water and steam reach saturation at the steam partial pressure. Vapor temperature would eventually stop rising as relative humidity approaches unity. With the gas region saturated, the warmer mixture exchanges heat with the colder water. Hence, the mixture temperature reverses direction and merges with the pool water temperature until it eventually reaches equilibrium. This discussion is depicted in the plots of pressure and temperatures for a system having a volume of 100 ft^3 and being at initial conditions of $P = 20 \text{ psia}$, $T = 200 \text{ F}$, and $\phi = 10 \%$. The initial water volume fraction (water volume divided by total volume) in this example is 3%.



Filling Rigid Volumes, Equilibrium Saturation Condition

In this case, we analyze a control volume initially at equilibrium state with specified initial pressure, temperature, relative humidity, and water volume fraction. Such a control volume may represent the suppression pool of a BWR, or the quench tank of a PWR. The role of such systems is to condense the injected mixture of water and steam. Although the injection, condensation, and subsequent pressurization of the control volume constitute a transient process, we only consider the initial and the final equilibrium states. The goal is to find the final pressure given the total mass and enthalpy of the injected mixture of water and steam (Figure IIc.4.2). Since the moist air is initially saturated and a saturated mixture is also injected into the control volume, then the moist air remains saturated throughout the event and the water in the pool also remains saturated at the steam partial pressure. To find the final pressure, we use the conservation equations of mass,

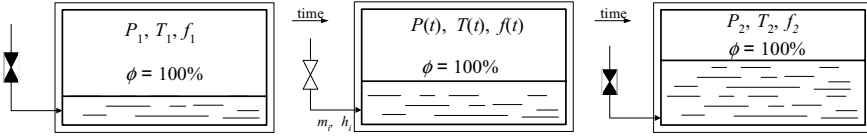


Figure IIc.4.2. A control volume representing a steam suppression system

energy, the equation of state, and the volume constraint. Mass balance for water and steam gives:

$$(m_{f1} + m_{g1}) + m_i = m_{f2} + m_{g2}$$

For energy balance, we use Equation IIa.8.2 as applied to the control volume, assuming a constant h_i :

$$(m_{f1}u_{f1} + m_{g1}u_{g1}) + (m_a u_{a1}) + m_i h_i = (m_{f2}u_{f2} + m_{g2}u_{g2}) + (m_a u_{a2})$$

Finally, the volume constraint gives:

$$m_{f2}v_{f2} + m_{g2}v_{g2} = V$$

There are four unknowns: T_2 , v_2 (u_2), m_{f2} , and m_{g2} . There are also four equations, three of which are listed above and the fourth is the equation of state. We begin solving the above set by eliminating m_{g2} from mass and volume constraint to find $m_{f2} = [(m_{f1} + m_{g1} + m_i)v_{g2} - V] / v_{fg2}$. Hence,

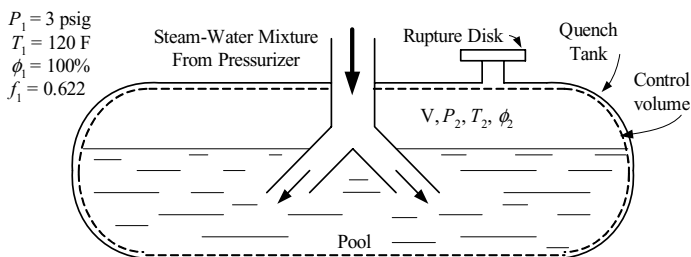
$$m_{g2} = [V - (m_{f1} + m_{g1} + m_i)v_{f2}] / v_{fg2}.$$

Substituting into the energy equation, we get:

$$C_1[(v_{g2} / v_{fg2})u_{f2} - (v_{f2} / v_{fg2})u_{g2}] + V(u_{fg2} / v_{fg2}) + C_2(T_2 - T_1) = C_3 \quad \text{IIc.4.1}$$

where C_1 , C_2 , and C_3 are constants given as $C_1 = m_{f1} + m_{g1} + m_i$, $C_2 = m_a c_{va}$, and $C_3 = (m_{f1}u_{f1} + m_{g1}u_{g1}) + m_i h_i$, respectively. We may substitute for $v_{f2} = v_f(T_2)$, $v_g = v_g(T_2)$, and other thermodynamic properties in Equation IIc.4.1. This would result in a non-linear algebraic equation, that is only a function of T_2 and can be solved by the Newton-Raphson method. Alternatively, we may assume a value for T_2 and iterate with the steam tables.

Example IIc.4.2. The quench tank of a PWR, as shown in the figure, has a volume of 217 ft³ (6 m³). Initial pressure, temperature, relative humidity, and water volume fraction (f_1) are specified in the figure. During an event, a total of 536 lbm (243 kg) of steam at an average enthalpy of 1133 Btu/lbm (2635.3 kJ/kg) enters the pool. The rupture disk will fail at a pressure of 145 psia (≈ 1 MPa). Find whether the disk remains intact or if it fails.



Solution: We first find the initial masses and internal energies as follows:

$$m_{f1} = f_1 V / v_{f1} = 0.622(217/0.01620) = 8331.3 \text{ lbm (3779 kg)}$$

Since $P_{v1} = \phi_1 P_g(T_1) = 1.6927 \text{ psia}$ hence, $P_{a1} = P_1 - P_{v1} = 17.70 - 1.6927 = 16 \text{ psia}$. We can now find the mass of air in the tank from:

$$m_a = P_a V_a / (R_a / M_a) T_1 = (16 \times 144) \times 82 / [(1545/28.97) \times (460 + 120)] = 6.11 \text{ lbm (2.77 kg)}$$

To find the mass of vapor we may either use $m_{v1} = V_a / v_{g1} = 82 / 203.26 = 0.403 \text{ lbm}$ or use the definition of the humidity ratio $m_{v1} = \omega_1 m_a$ with $\omega_1 = 0.622 P_{v1} / (P - P_{v1}) = 0.622 \times 1.6927 / (17.7 - 1.6927) = 0.0657$ to get $m_{v1} = \omega_1 m_a = 0.0657 \times 6.11 = 0.402 \text{ lbm}$. Finally, $u_{f1} = 87.97 \text{ Btu/lbm}$ and $u_{g1} = 1113.6 \text{ Btu/lbm}$. Thus $C_1 = 8331.3 + 0.403 + 536 = 8868 \text{ lbm (4022.5 kg)}$

$$C_2 = 6.11(0.171) = 1.045 \text{ Btu/F (1.984 J/C)}$$

$$C_3 = (8331.3 \times 87.97 + 0.403 \times 1113.6 + 536 \times 1133) = 1.341\text{E6 Btu (1414.8 MJ)}$$

Upon substitution into Equation Ilc.4.1, we get:

$$\{8868[(v_{g2}/v_{fg2})u_{f2} - (v_{f2}/v_{fg2})u_{g2}] + 217(u_{fg2}/v_{fg2}) + 1.045(T_2 - 120)\} - \{1.3406\text{E6}\} = 0$$

To solve this equation iteratively with steam tables, we guess a T_2 , say $T_2 = 250 \text{ F}$. From the steam tables, we find $v_{f2} = 0.01787 \text{ ft}^3/\text{lbm}$, $v_{g2} = 3.7875 \text{ ft}^3/\text{lbm}$, $v_{fg2} = 3.7697 \text{ ft}^3/\text{lbm}$, $u_{f2} = 311.3 \text{ Btu/lbm}$, and $u_{g2} = 1190.1 \text{ Btu/lbm}$

The answer converges to $T_2 = 182.5 \text{ F}$ after 6 trials as shown below.

T_2 (F)	v_{f2} (ft ³ /lbm)	v_{fg2} (ft ³ /lbm)	v_{g2} (ft ³ /lbm)	u_{f2} (Btu/lbm)	u_{fg2} (Btu/lbm)	u_{g2} (Btu/lbm)	Residue
340	0.01787	3.76970	3.78750	311.3	878.8	1190.1	112
250	0.01701	3.8020	3.8190	218.7	868.7	1087.4	67.6
200	0.01664	3.8220	3.839	168.1	905.5	1073.4	11.3
160	0.01639	3.82700	3.841	127.8	934.2	1062.1	-22.5
182	0.01652	3.81720	3.833	149.8	918.5	1068.3	-1.13

The moist air volume becomes

$$V_{a2} = 217 - (135 + 536 \times 0.01652) = 73.15 \text{ ft}^3 (2.07 \text{ m}^3)$$

At $T_2 = 182.5 \text{ F}$, tank pressure is $P_2 = P_{g2}(T_2) + P_{a2}(T_2) = 7.94 + [6.11 (1454/28.97) (182.5 + 460) / 73.15] = 7.94 + 11.57 = 19.5 \text{ psia (0.134 MPa)}$.

This pressure is too low to cause the failure of the rupture disk. However, we assumed that all the steam is condensed in the pool. Pressure rises substantially even if a small fraction of steam is not condensed and escapes to the moist region above the pool. This is shown in Example IIc.4.3.

Filling Rigid Volumes, Equilibrium Saturation Condition, Alternate Solution

To avoid iteration, we may choose an approximate solution for problems similar to Example IIc.4.2. In this method, we ignore the presence of air (and other non-condensable gases) in the vapor region. This is a valid assumption only if a small amount of air exists in the volume. We then use a “lumped parameter” approach in which the mixture of the pool water and the moist air is assumed to be mixed homogeneously. We also assume that the incoming mixture of water and steam mixes instantaneously and perfectly with the content of the control volume. The mass balance for water gives:

$$m_1 + m_i = m_2$$

The volume constraint gives:

$$v_{f2} + x_2 v_{fg2} = V/m_2$$

and the energy equation for the mixture becomes:

$$m_2(u_{f2} + x_2 u_{fg2}) = (m_1 u_1 + m_i h_i)$$

Substituting for m_2 from the mass balance and eliminating x_2 between the energy equation and the volume constraint, we find an equation equivalent to Equation IIc.4.1:

$$u_{f2} + (V/m_2 - v_{f2})u_{fg2}/v_{fg2} = (m_1 u_1 + m_i h_i)/(m_1 + m_i) \quad \text{IIc.4.2}$$

Solving Equation IIc.4.2 for $P_{sat}(T_2)$, we then find $P_2 = P_{sat}(T_2) + m_{air}RT/V$.

Filling Rigid Volumes, Equilibrium Superheated Condition

In the previous section we considered control volumes in which water vapor remains saturated throughout the charging process. We now consider cases in which water vapor is superheated steam at the final state. The solution procedure is similar to the derivation for the saturation condition however, unlike the saturation condition in this case, pressure is not a function of temperature and has to be calculated separately. An example for such cases includes a main steam line break inside the containment building of a PWR and subsequent pressurization of the containment. From the volume constraint we have:

$$v_2 = V/(m_1 + m_i) = A_1 \quad \text{IIc.4.3}$$

The energy balance for the control volume, assuming no heat transfer to or from the control volume, yields:

$$(m_1 + m_i) u_2 + m_a c_{va}(T_2 - T_1) = m_1 u_1 + m_i h_i$$

This equation may be written as:

$$u_2 + A_2(T_2 - T_1) = A_3 \quad \text{IIC.4.4}$$

where $A_2 = m_a c_{va}/(m_1 + m_i)$ and $A_3 = (m_1 u_1 + m_i h_i)/(m_1 + m_i)$. Equations IIC.4.3, IIC.4.4, and the equation of state provide three equations for three unknowns P_2 , T_2 , and v_{v2} (u_{v2}). Since iteration on both P_2 and T_2 is very laborious, we treat the vapor as an ideal gas and find the vapor pressure from $P_{v2} = m_{v2} R_v T_2 / V$. In this approach, we have implicitly accounted for and hence, superseded the volume constraint for vapor. Having P_{v2} and T_2 , we read u_2 from the steam tables and compare it with u_2 calculated from Equation IIC.4.4. We continue this iterative process until the convergence criterion is met. We then find the final pressure from:

$$P_2 = \frac{m_a R_a T_2}{V} + \frac{m_2 R_v T_2}{V} \quad \text{IIC.4.5}$$

Example IIC.4.3. An initially drained quench tank contains moist air at 120 F (48.9 C) at a relative humidity of 50%. A total of 54 lbm (24.5 kg) of steam at an average enthalpy of 1133 Btu/lb (2635.27 kJ/kg) enters the tank. Find the tank final temperature and pressure. Tank has a volume of 217 ft³ ($\approx 6 \text{ m}^3$).

Solution: In this case, moist air occupies the entire volume of the tank. Initial mass of vapor in the tank is found from $P_v = 0.5(1.6927) = 0.85 \text{ psia}$. Therefore, $v_1 = (0.85 \times 120) = 405.5 \text{ ft}^3/\text{lbm}$ and $m_1 = 217/405.5 = 0.535 \text{ lbm}$. Thus, $m_2 = 0.535 + 54 = 54.535 \text{ lbm}$. Again, from the steam tables, $u_1 = 1049.14 \text{ Btu/lbm}$ and:

$$A_3 = (0.535 \times 1049.14 + 54 \times 1133)/54.535 = 1132.17 \text{ Btu (1194.5 kJ/kg)}$$

To find air mass, we calculate $P_{a1} = P_1 - P_{v1} = 17.70 - 0.85 = 16.85 \text{ psia}$. Therefore, the mass of air in the tank is:

$$m_a = P_a V_a / (R_a / M_a) T_1 = (16.85 \times 144) \times 217 / [(1545/28.97) \times (460 + 120)] = 17 \text{ lbm (7.71 kg)}.$$

Hence, $A_2 = 17(0.171)/m_2 = 0.053$ and Equation IIC.4.4 becomes $u_2 + 0.053(T_2 - 120) = 1132.17$.

We start the iteration process by guessing $T_2 = 400 \text{ F}$.

$$P_{v2} = 54.535(1545/18)(460 + 400)/217 = 128.83 \text{ psia giving } u_2 = 1132.33 \text{ Btu/lbm.}$$

From Equation IIC.4.4 for $u_2 + 0.053(T_2 - 120) = 1132.17$ we find $u_2 = 1132.17 - 0.053(400 - 120) = 1117.24 \text{ Btu/lbm}$. The trials are tabulated as follows:

T_2 (F)	P_{v2} (psia)	$(u_2)_{\text{Table}}$ (Btu/lbm)	$(u_2)_{\text{Energy Eq.}}$ (Btu/lbm)
400	128.8	1132.3	1117.24
350	121.3	1111.1	1119.98
355	122.1	1113.3	1119.97
370	124.3	1119.7	1118.92
368	124.0	1118.9	1119.02

Having the final equilibrium temperature at about $T_2 = 368.5 \text{ F}$, final pressure becomes $P_2 = 124 + [17 \times (1545/28.97) \times (460 + 368.5) / (217 \times 144)] = 124 + 24 = 148 \text{ psia (1.02 MPa)}$. This indicates that the quench tank rupture disk of Example IIC.4.3 fails during this event.

Thermal Design of Cooling Towers

Cooling towers are ultimate heat sinks. They are used in power production and other applications such as production of chilled water. Cooling towers are used when naturally occurring heat sinks such as lakes and other large bodies of water are not available or are available but the flow of water is not sufficient to comply with regulations for prevention of thermal pollution. Cooling towers for power production are either of induced draft or of natural draft type. Cooling towers may also be of wet or of dry type. In the dry cooling tower, atmospheric air passes through tubes carrying turbine exhaust. Hence, in dry cooling towers, the only means of transferring heat to the atmospheric air is through sensible heat. In the wet cooling towers as shown in Figure IIc.4.3, the circulating water cooling the turbine exhaust is sprayed inside the tower and is cooled by both sensible and latent heat removal due to the counter current flow of atmospheric air drawn into the cooling tower. The packing facilitates contact between the warmer sprayed water and the colder atmospheric air hence, increasing the rate of heat transfer. It also causes breakup of water droplets to enhance evaporation. The energy for evaporation is supplied by the warmer, sprayed water at 1. As a result, water exiting at 2 is cooler than the water sprayed at 1. The evaporation also causes the moist air exiting the tower at 4 to be near or at saturation. The makeup water that flows into the tower at 5 is meant to compensate for the loss of water through evaporation.

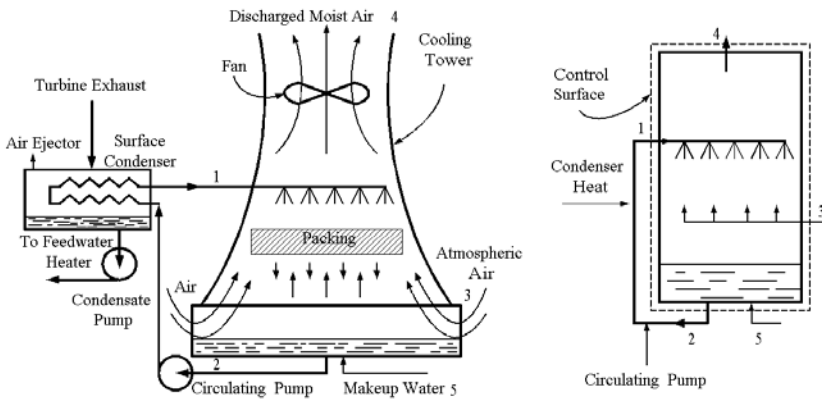


Figure IIc.4.3. A wet, induced-draft cooling tower

Conservation Equations for Wet Cooling Towers

Let's consider the control volume representing the ideal wet cooling tower. The streams entering the control volume include atmospheric air, warm circulating water, and makeup water. The streams leaving the control volume include the nearly saturated moist air and colder water. Thermal analysis of the cooling tower is based on two conservation equations of mass, one for air and one for water, and one energy equation for the mixture of air and water. For steady state operation, mass balance for air gives:

$$\dot{m}_{a3} = \dot{m}_{a4} = \dot{m}_a$$

Mass balance for water gives:

$$\dot{m}_1 + \dot{m}_{w3} + \dot{m}_5 = \dot{m}_2 + \dot{m}_{w4}$$

Since $\dot{m}_1 = \dot{m}_2$, then:

$$\dot{m}_{w3} + \dot{m}_5 = \dot{m}_{w4}$$

We now solve for the mass flow rate of the make up water in terms of the differential rate of moisture content at the outlet and inlet to the control volume:

$$\dot{m}_5 = \dot{m}_{w4} - \dot{m}_{w3} = (\omega_4 - \omega_3)\dot{m}_a \quad \text{IIC.4.6}$$

The energy balance gives:

$$\dot{m}_1 h_1 + (\dot{m}_{w3} h_{w3} + \dot{m}_a c_{pa} T_3) + \dot{m}_5 h_5 = \dot{m}_2 h_2 + (\dot{m}_{w4} h_{w4} + \dot{m}_a c_{pa} T_4) \quad \text{IIC.4.7}$$

We may now approximate the enthalpies of the moisture content of the incoming and exiting streams of moist air as saturated enthalpies at the specified temperatures. After simplifications, substitutions, and rearrangement of Equation IIC.4.7, we find the required mass flow rate of air as:

$$\dot{m}_a = \frac{\dot{m}_1 (h_1 - h_2)}{(\omega_4 h_{g4} - \omega_3 h_{g3}) - (\omega_4 - \omega_3) h_5 + c_{pa} (T_4 - T_3)} \quad \text{IIC.4.8}$$

Having the mass flow rate of air from Equation IIC.4.8, we can choose the fan for induced-draft tower or size the tower for natural-draft cooling towers. The mass flow rate of makeup water is also found (Equation IIC.4.6) based on the mass flow rate of air.

Example IIC.4.4. The condenser of a power plant is cooled by a circulating water flow rate of 100×10^6 lbm/h. The circulating water enters the cooling tower at 110 F and leaves the tower for the condenser at 95 F. Atmospheric air enters the tower at 75 F and 35% relative humidity. Moist air leaves the tower at 90 F and 95% relative humidity. The make up water enters the tower at 70 F. Find the mass flow rate of air and the make up water.

Solution: We first find the pertinent thermodynamic properties:

T (F)	P (psia)	h_f (Btu/lbm)	h_g (Btu/lbm)
70	-	38.05	1092.1
75	0.43	43.05	1094.3
90	0.69	58.02	1100.8
95	-	63.01	1102.9
110	-	77.98	1109.3

Having vapor pressure and relative humidity, we can find the humidity ratios. For air entering the tower $P_{v3} = 0.35(0.43) = 0.15$ psia. For air leaving the tower, $P_{v5} = 0.95(0.69) = 0.65$ psia. Therefore,

$$\omega_3 = 0.622(0.15)/(14.7 - 0.15) = 0.00641. \text{ Similarly,}$$

$$\omega_4 = 0.622(0.65)/(14.7 - 0.65) = 0.0287.$$

Substituting in Equation Ilc.4.8, we get:

$$\dot{m}_a = \frac{1E8(77.98 - 63.01)}{(0.0287 \times 1100.8 - 0.0064 \times 1094.3) - (0.0287 - 0.0064)38 + 0.171(90 - 75)} \approx 57E6 \text{ lbm/hr}$$

Substituting in Equation Ilc.4.6, we find

$$\dot{m}_5 = (0.0287 - 0.0064)57E6 \approx 1.3E6 \text{ lbm/hr.}$$

Thermal Design of Containment

The containment building is the last barrier against release of radioactive materials to the environment in the case of a hypothetical accident. There are several types of containments, the design of which depends on the type of the nuclear reactor and the architect engineer. For example, to deal with thermalhydraulic loads, BWR containments are equipped with a suppression pool while some types of PWR containment utilize large blocks of ice. Figure Ilc.4.4 shows the schematic of a PWR *large, dry containment*. With respect to thermalhydraulic loads, PWR containments should withstand the consequences of two types of postulated accidents; a loss of coolant accident (LOCA) and a main steam line break (MSLB). A LOCA refers to a primary side pipe break of the hot or the cold leg, such as a double-ended guillotine break at location a-a in Figure Ilc.4.4, for example. A MSLB refers to rupture of the main steam line inside the containment such as a double-ended guillotine break at break location b-b. The LOCA and MSLB are referred to as design basis accidents.

The PWR containment analysis for both LOCA and MSLB requires two key inputs, mass flow rate and enthalpy of the fluid flowing through the break into the containment. We analyze containment in both the design phase and during operation. In the design phase, our intention is to find the free volume, that can accommodate the mass and energy transfer so that the peak pressure and temperature are kept below the specified design limits. During operation, containment analysis is required subsequent to any modification that may impact the containment response to above postulated accidents. During plant operation, we therefore seek containment peak pressure and temperature for given free volume.

A containment building, or simply containment, is generally equipped with active safety systems such as spray and air coolers to provide a heat sink in the case of an accident. The containment structure and internals also absorb a substantial amount of energy during an accident, thus they are referred to as *passive heat sinks*. In the case of the containment structure, some heat is also transferred to the surroundings through the primer, paint, steel liner plate, and the one meter thick concrete wall. The heat source for the containment depends on the postulated accident and the reactor type. For a LOCA, the heat source includes the latent heat

of the primary-system inventory, the sensible heat stored in the reactor system metals, and the decay heat of the fission fragments. For a MSLB in a PWR, the heat source includes the latent heat of water inventory of the secondary side, the sensible heat from the stored energy in the steam generator metals, and the heat transfer from the primary side through the tubes. Additionally, some exothermic chemical reactions, such as zirconium reacting with water at high temperatures, add to the containment thermal load. Handling the hydrogen produced in such chemical reactions is another constraint for the design of the containment. This discussion is summarized in Table IIc.4.1.

Below we perform a containment response analysis for both LOCA and MSLB to find peak pressure and temperature for a PWR large dry containment. Similar analysis exists for a BWR containment.

Table IIc.4.1. Factors affecting PWR containment response to accidents

Event	Heat Source	Heat Sink	Source of Emergency Cooling
LOCA	Decay heat*	Containment spray	High-pressure safety injection
	Coolant internal energy	Containment air coolers	Low pressure safety injection
	Metal Stored energy	Passive heat sinks	Safety water tanks
	Reactor pump heat		
	Exothermic reactions		
MSLB	Latent heat of coolant	Containment spray	Auxiliary feedwater
	Stored energy	Containment air coolers	
	Exothermic reactions	Passive heat sinks	

* See description in Chapter VIe.

Case A: Containment Response Analysis to LOCA in PWRs

In this case, we seek peak pressure for given containment volume. To obtain pressure and temperature versus time, we need to have the mass flow rate and enthalpy of the flow at the break as a function of time. We leave this rigorous treatment of containment analysis to Chapter VI d. For now, we include the primary side of the reactor in the containment control volume (Figure IIc.4.5). For this control volume, there is no flow entering or exiting and no shaft or expansion work. We then find containment peak pressure by integrating the simplified form of Equation IIa.8.1 from the initial to the final state. The initial state refers to the primary system being intact. The final state refers to a condition at which the primary side has discharged most of its inventory to the containment and has reached thermal equilibrium with the containment.

In the analysis that follows, m_{v1} is the initial mass of water vapor in the containment atmosphere, m_{w1} is the initial mass of water in the primary-system. Similarly, V_{v1} is the free volume of the containment (according to the Dalton’s model, $V_{v1} = V_a$) and V_{w1} is the volume of the primary-system. Finally, m_{w2} is the total mass of water and steam in $V_{v1} + V_{w1}$. Similar subscripts are used for the internal energy terms.

$$m_a u_{a1} + m_{v1} u_{v1} + m_{w1} u_{w1} + Q_{Decay} + Q_{Metal} + Q_{Pump} - Q_{Spray} - Q_{Cooler} - Q_{Structure} \\ = m_a u_{a2} + m_{w2} u_{w2}$$

To maximize the energy transfer to the containment atmosphere, we drop all energy removal terms due to the action of spray and air cooler, as well as the heat absorption in the containment structure:

$$m_a u_{a1} + m_{v1} u_{v1} + m_{w1} u_{w1} + Q_{Decay} + Q_{Metal} + Q_{Pump} = m_a u_{a2} + m_{w2} u_{w2} \quad \text{IIc.4.10}$$

We have three unknowns, m_{w2} , u_{w2} , and T_2 . To complete the set, we use the volume constraint:

$$v_{w2} = (V_{w1} + V_{v1})/m_{w2} \quad \text{IIc.4.11}$$

Since the mass transfer in a LOCA from the primary side to the containment is primarily in the form of a two-phase mixture, we expect that the containment atmosphere becomes saturated in steam, yielding $\phi_2 = 1$. To solve the above set of three equations, we substitute for m_{w2} from the continuity equation into the energy equation and the volume constraint. We then assume a steam quality x_2 , and iterate on T_2 between the two equations and the steam tables.

Example IIc.4.5. The primary side of a PWR has a volume of 11,000 ft³ (311.5 m³). The reactor is operating at an average pressure of 2200 psia (15 MPa) and average temperature of 575 F (302 C). Containment initial conditions are given as 16.5 psia (114 kPa), 125 F (52 C), and 20% relative humidity. Containment volume is 2E6 ft³ (56,636 m³). Find final equilibrium pressure following a LOCA.

Solution: For containment, we first find the initial steam partial pressure:

$$P_{v1} = \phi P_g(125) = 0.2 \times 1.9424 = 0.388 \text{ psia (2.67 kPa)}$$

We now find initial masses and energies for which we first find the thermodynamic properties as follows:

P (psia)	T (F)	$P_g(T)$ (psia)	v (ft ³ /lbm)	u (Btu/lbm)
0.388	125	1.9424	898	1052.4
2250	575	—	0.0221	569.84

$m_{v1} = V_{v1}/v_1 = 2,000,000/898 = 2227 \text{ lbm}$. The air mass is found from $m_a = P_{a1}V_d/(R_aT_1)$. Since air pressure is $P_{a1} = P_1 - P_{v1} = 16.5 - 0.388 = 16.11 \text{ psia}$:

$$m_a = \frac{P_{a1}V_{a1}}{R_aT_1} = \frac{(16.11 \times 144) \times (2 \times 10^6)}{(1545/28.97)(460 + 125)} = 150015 \text{ lbm (68,047 kg)}$$

We now find the mass of water in the primary system as $m_{w1} = V_{w1}/v_{w1} = 11,000/0.0221 = 497737$ lbm. Hence,

$$m_{w2} = m_{v1} + m_{w1} = 2227 + 497737 = 499,964 \text{ lbm.}$$

From Equation Ilc.4.11 we find $v_2 = (2,000,00 + 11,000)/499,964 = 4 \text{ ft}^3/\text{lbm}$. Having v_2 , we guess a value for T_2 and read v_{f2} and v_{g2} from the steam tables to find x_2 as $x_2 = (v_2 - v_f)/v_{fg}$.

$$T_2 = T_1 + \frac{(m_{v1}u_{v1} + m_{w1}u_{w1}) - m_{w2}u_{w2}}{m_a c_{va}}$$

where we have only considered the coolant internal energy (see the comment below). We summarize the data we have found so far:

m_a lbm	m_{v1} lbm	m_{w1} lbm	u_{v1} Btu/lbm	u_{w1} Btu/lbm	m_{w2} lbm	v_{w2} ft ³ /lbm
150015	2227	497737	1052.4	569.84	499,964	4

$$m_{v1}u_{v1} + m_{w1}u_{w1} = 2227(1052.4) + 497737(569.84) = 2.859\text{E}8 \text{ Btu (3E8 kJ)}$$

We begin the iteration process by assuming a value for T_2 and find an updated value for T_2 as follows:

T_2 (F)	v_f (ft ³ /lbm)	v_{fg} (ft ³ /lbm)	u_f (Btu/lbm)	u_{fg} (Btu/lbm)	x_2 (-)	u_{w2} (Btu/lbm)	T_2 (F)
268.10	0.0172	10.3941	236.90	854.62	0.383	564.4	270

Since $\varepsilon = (270 - 268.10)/268.10 = 0.7\%$, we use 270 F as a reasonably accurate final temperature. Having final equilibrium temperature, we find:

$$P_{w2} = P_g(268.1 \text{ F}) = 40.73 \text{ psia (281 kPa)}$$

$$P_{a2} = m_a R_a T_2 / (V_{w1} + V_{v1}) = 150015 \times 53.33(268.1 + 460) / 2,011,000 = 8.68 \text{ psia (59.8 kPa)}$$

Therefore, final equilibrium pressure is $P_2 = P_{w2} + P_{a2} = 40.73 + 8.68 = 49.4 \text{ psia (341 kPa)}$.

Comment: In this solution, we only accounted for the internal energy of the primary side coolant and did not consider the heat addition from all other sources. If the pumps are tripped, their contribution to the energy equation is eliminated. However, inclusion of the decay heat and the sensible heat of the reactor structure and its internals (metal stored energy) requires detailed knowledge of the system. If we assume that the contribution from the decay heat and stored energy in the above example is $Q_{Total} = 25\text{E}6 \text{ Btu}$, we may follow the same steps outlined above to find $T_2 = 276 \text{ F}$ and $P_2 = 64.7 \text{ psia}$. The method of obtaining the pressure and temperature trends for containment response is discussed in Chapter VI d.

Case B: Containment Response Analysis to MSLB in PWRs

In this case, we also seek the final equilibrium pressure for given containment volume, where the initial state refers to the steam generator being intact. The final state refers to thermal equilibrium between the broken steam generator and the containment. The control volume we choose consists of the containment and the

broken steam generator (Figure Ilc.4.6). As explained in Case A, the selection of such a control volume eliminates the need for having the break mass flow rate and enthalpy versus time. On the other hand, such treatment precludes us from predicting the trend of the containment pressure and temperature during the event. The same sets of equations we developed for LOCA are also applicable here. However, in a MSLB, mass transfer from the broken steam generator to the containment is primarily in the form of dry steam. Therefore, we expect that the containment atmosphere becomes superheated in steam, yielding $\phi_2 < 1$.

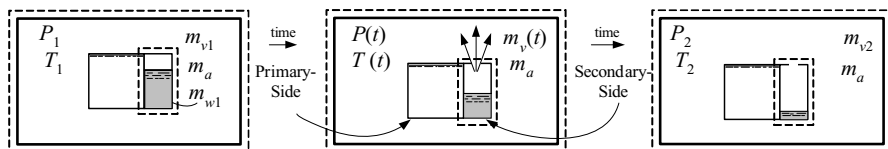


Figure Ilc.4.6. Depiction of a MSLB in a PWR containment

In this case, like Case A, we use the Dalton law of partial pressures for volume constraint. However, we calculate the partial pressure of the superheated steam from the ideal gas law, per Equation Ilc.4.5. This is a reasonable approximation in the range of interest for pressure and temperature:

$$P_2 = m_a R_a T_2 / (V_{v1} + V_{w1}) + m_{v2} R_v T_2 / (V_{v1} + V_{w1}) \quad \text{Ilc.4.12}$$

Example Ilc.4.6. The secondary side of a PWR steam generator has a volume of 227 m³ of which 75 m³ is water. Steam generator pressure is 6.21 MPa. Initial containment pressure, temperature, and relative humidity are 0.101 MPa, 50 C, and 50%, respectively. Containment volume is 56,636 m³. Find the final equilibrium pressure and temperature following a MSLB. The amount of heat transferred from the primary side to the secondary side is 897.6E8 J.

Solution: The initial vapor mass in the containment is found from

$$P_{v1} = \phi P_g(50 \text{ C}) = 0.5 \times 0.0123 = 0.00615 \text{ MPa}$$

We can find vapor mass from either the steam tables or the ideal gas law. From the steam tables

$$v(0.00615 \text{ MPa} \text{ \& } 50 \text{ C}) = 24.7 \text{ m}^3/\text{kg}.$$

Hence, the vapor mass is found as:

$$m_{v1} = 56,636 / 24.7 = 2293 \text{ kg}$$

From the ideal gas law, $m_{v1} = 6150 \times 56,636 / [(8314/18) \times (273 + 50)] = 2335 \text{ kg}$. The error in the calculation of vapor mass by using the ideal gas law is less than 2%. Similarly, for air mass

$$m_a = (101000 - 6150) \times 56,636 / [(8314/28.97) \times (273 + 50)] = 58,000 \text{ kg}$$

The initial mass of water and steam in the secondary side of the steam generator can be calculated from the specific volumes, found from the steam tables. For water, $m_{f1} = 75/0.0013 = 57,692$ kg. For steam, $m_{g1} = (227 - 75)/1.69 = 4,841$ kg. Therefore, $m_{w1} = 4841 + 57,692 = 62,533$ kg. We also find $x_1 = m_{\text{steam}}/m_{\text{water}} = 4841/62,533 = 0.077$. Therefore,

$$u_{w1} = 1216.75 + 0.077 \times 1370.79 = 1322.86 \text{ kJ/kg.}$$

To summarize;

m_a (kg)	m_{v1} (kg)	m_{w1} (kg)	u_{v1} (kJ/kg)	u_{w1} (kJ/kg)	m_{v2} (kg)
58,000	2293	62,533	2441.36	1323	64,826

We now guess T_2 , and find P_{v2} , from Equation IIc.4.12. Assuming $T_2 = 210$ C we find P_{v2} :

$$P_{v2} = [67826 \times 462 \times 483/56863] = 0.254 \text{ MPa}$$

We use the calculated P_{v2} and the assumed T_2 to find u_{v2} from the steam tables as $u_{w2} = 2,667$ kJ/kg. Having the final internal energies, the final temperature can be found from Equation IIc.4.10:

$$T_2 = T_1 + \frac{(m_{v1}u_{v1} + m_{w1}u_{w1} + Q_{\text{Primary-Secondary}}) - m_{w2}u_{w2}}{m_a c_{va}}$$

Substituting values for masses, internal energies, and heat transfer between primary and the secondary:

$$T_2 = 50 + \frac{(2293 \times 2441.36 + 62,533 \times 1323 + 0.898\text{E}8) - 64,826 \times 2.667}{58000 \times 0.713} = 176 \text{ C}$$

We continue the iteration until the convergence criterion is met. The final answer is $T_2 = 200$ C and $P_2 = 0.393$ MPa.

Comment: This is a useful method to find the final state inside the containment following a MSLB (or a LOCA). In practice, the conservation equations of mass and energy are integrated over a small time step to obtain u and v . The corresponding P and T are found in a pressure search process. This process is repeated until the end of the specified duration is reached.

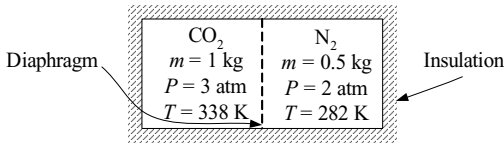
QUESTIONS

- What is the difference between the Dalton and the Amagat model?
- Apply the Amagat model to Example IIc.1.2. What conclusion do you reach?
- In a containment of a nuclear plant, the relative humidity is measured as 35% and in the containment of another plant, it is measured as 70%. If both containments are at the same temperature and total pressure, which containment has higher steam pressure? If both containments have also equal volumes, which containment has higher air mass?
- Consider an unsaturated moist air at total pressure P , temperature T , and relative humidity ϕ . What is the significance of $P_g(T)$ and of $T_g(P_v)$?

- Is the dew point temperature reached in an isochoric or an isobaric process?
- Describe the humidification process.
- Consider a fully insulated system consisting of two regions. The first region contains water at P_1 and T_1 . This region is separated from the second region by a thermally conducting membrane. The second region contains moist air also at P_1 and T_1 with $\phi = \phi_1 < 1$. The membrane is now removed and $\phi_2 = 1$. Is $P_2 > P_1$?
- Why is there a need for makeup water in the operation of cooling towers?
- Consider the large dry containment of a PWR. There are many internals in the containment such as pedestals, pipe supports, polar crane, stairways, etc. The obvious disadvantage of the containment internals is to reduce the free volume of the containment. From a thermodynamic point of view, what is the advantage of having the internals in the containment during a design basis accident?
- With respect to containment response, what are the two major differences between a LOCA and a MSLB?

PROBLEMS

1. A tank, having a volume of 5 m^3 , is filled with N_2 and 2 kg of CO_2 at a pressure and temperature of 150 kPa and 50 C, respectively. Find the partial volumes according to the Amagat and the partial pressures according to the Dalton model.
2. A rigid tank contains 1 kg of nitrogen at 38 C and 2 MPa. We now add oxygen to the tank in an isothermal process until the pressure in the tank reaches 2.76 MPa. Find the mass of oxygen that entered the tank in this process. [Ans.: 0.35 kg].
3. Consider a room having a volume of 75 m^3 maintained at $P = 1 \text{ atm}$, $T = 25 \text{ F}$ and $\phi = 70\%$. a) Use the Dalton model to find the partial pressures of air and water vapor. b) Use the Amagat model to find the partial volumes of air and water vapor. What conclusion do you reach about the applicability of the Amagat model to water vapor? [Hint: In case b, find the state of the water vapor from the steam tables by having its pressure and temperature].
4. A large dry containment of a PWR has a volume of $2\text{E}6 \text{ ft}^3$. At normal operation, the mixture of air and superheated steam is at a total pressure of 1.8 psig (16.5 psia) and temperature of 125 F. The relative humidity in the containment is measured as 30%. Find the masses of air and steam in the containment.
5. A cylinder contains 0.8 lbm of CO_2 and 0.5 lbm of N_2 at 18 psia and 80 F. In a polytropic process ($n = 1.25$), the content is compressed to 65 psia. Find final temperature, the work, the heat transfer, and the change in the mixture entropy.
6. Consider two well insulated-tanks as shown in the figure. The tanks contain carbon dioxide and nitrogen at the given pressures and temperatures. The thermally non-conductive diaphragm is now removed. Find the final temperature and pressure of the mixture at equilibrium.



[Hint: Since the tanks are well insulated, there is no heat transfer with the surroundings. Since the boundaries are fixed, there is no work. Hence, from the first law, $U_2 = U_1$].

7. Two steady flow streams of gases at different pressure and temperature are merged into one stream in an adiabatic process. Use the given data to find the temperature of the merged streams. Data: One stream consists of 2.3 kg of nitrogen at 103.5 kPa and 150 C. The other stream consists of 1 kg of CO_2 at 138 kPa and 38 C. [Ans.: 120 C].

8. A cylinder contains gases with the following volumetric analysis: 13% CO_2 , 12% O_2 , and 75% N_2 . Find c_p , c_v , and R for this mixture of gases. Specific heat at constant pressure of CO_2 , O_2 , and N_2 are given as 1.271 kJ/kg K, 1.11 kJ/kg K, and 1.196 kJ/kg K, respectively.

9. Find the dew point temperature for an unsaturated moist air at $P = 15 \text{ psia}$, $T = 120 \text{ F}$, and $\phi = 30\%$. [Ans: $T_{\text{Dew Point}} = T_g(P_v) \approx 80 \text{ F}$].

10. Find an expression for relative humidity in terms of ω

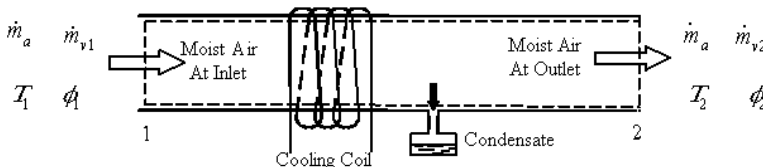
[Ans: $\phi = (\omega/0.622)\{P_a/P_g(T_v)\}$ or alternatively $\phi = \{\omega/(0.622 + \omega)\}\{P/P_g(T_v)\}$].

11. Find the mass fraction of water vapor in moist air at 30 psia, 200 F, and 65%.

12. Consider 5-lbm sample of moist air initially at 20 psia, 150 F, and 50% relative humidity. This mixture is cooled at constant pressure to 70 F. Find a) the humidity ratio at state 1, b) the dew point temperature at states 1 and 2, and c) the amount of condensate at state 2.

13. A rigid tank contains 0.5 kg of moist air at 1.034 MPa, 160 C, and $\phi = 100\%$. We now cool the tank until its temperature drops to 82 C. Find a) the amount of heat removed and b) the amount of condensate produced in this process. [Ans.: - 914 kJ and 0.4 kg].

14. Moist air at 15 psia, 90 F, and a relative humidity of 60% enters a cooling duct at a rate of 1200 ft³/m. Temperature of the saturated mixture at the exit of the cooling coil is 65 F. Assuming negligible pressure drop, find the mass flow rate of the condensate produced in the cooling duct.



15. Determine the amount of condensate, the final pressure, and heat transfer in the cooldown process of a sample of moist air. The process takes place in a rigid container having a volume of 35 m^3 . Moist air is initially at 1.5 bar, 120 C, and 10%. The final temperature is 22 C. [Ans.: 3.15 kg].

16. Consider constant-volume cooldown of a mixture of water vapor and nitrogen in a 17.66 ft^3 container. The mixture is originally at 122 F, 290 psia, and 40% relative humidity. The mixture is cooled to 50 F. Find the heat transfer in this process. [$Q_{CV} = -321 \text{ Btu}$]

17. Find the relative humidity in a room at a temperature of 20 C. The wet bulb temperature is 15.5 C.

[Ans.: 63%].

18. In Example IIC.4.2, we assumed that all of the incoming steam is condensed in the quench tank. Find the final pressure assuming that 5% of the steam escapes from the pool region to the vapor region.

19. A power plant uses a cooling tower as the heat sink. The net power produced by the plant is 270 MWe. The plant thermal efficiency is 35%. Use these and other pertinent data given below to find a) the mass flow rate of air and b) the mass flow rate of make up water.

circulating water: inlet temperature $T_{wi} = 104 \text{ F}$ (40 C) and exit temperature $T_{we} = 86 \text{ F}$ (30 C)

air: inlet temperature $T_{ai} = 77 \text{ F}$ (25 C), relative humidity $\phi_i = 35\%$, air exit temperature $T_{ae} = 95 \text{ F}$ (35 C) and relative humidity $\phi_e = 90\%$

make-up water: inlet temperature $T_{mw} = 68 \text{ F}$ (20 C).

[Ans.: for air: $57.83\text{E}6 \text{ lbm/h}$, for makeup water: $1.366\text{E}6 \text{ lbm/h}$].

20. A BWR containment design is suggested as shown in the Figure. The reactor is isolated within a drywell compartment. A rupture disk caps the end of a duct leading into a vapor suppression pool of water. The pool is inside a secondary compartment. The rupture disk fails at a differential pressure of 60 psi (0.414 MPa). Now consider the case of a main steam line break. Use the following data to find the time that the rupture disk fails.

Drywell: initial temperature $T_i = 100 \text{ F}$ (38 C), initial pressure $P_i = 14.7 \text{ psia}$ (0.1013 MPa), initial relative humidity $\phi_i = 0\%$, and free volume $V_{drywell} = 5\text{E}5 \text{ ft}^3$ ($14.16\text{E}3 \text{ m}^3$).

Secondary-containment: initial pressure = 14.7 psia (0.1013 MPa), and free volume = $5\text{E}6 \text{ ft}^3$ ($14.16\text{E}4 \text{ m}^3$)

Suppression pool: water volume = $2.3\text{E}5 \text{ ft}^3$ ($6.5\text{E}3 \text{ m}^3$) and initial temperature = 100 F (38 C).

Steam blowdown: steam mass flow rate from the steam line to the drywell $\dot{m} = 500 \text{ lbm/s}$ (227 kg/s) for the duration of $t < 360 \text{ s}$ and $\dot{m} = 500e^{-t/\theta} \text{ lbm/s}$ ($227e^{-t/\theta} \text{ kg/s}$) for $t \geq 360 \text{ s}$ and $\theta = 100 \text{ s}$. In order to avoid the necessity of an iterative solution, use the following simplifying assumptions:

- a) the atmosphere components are uniformly mixed and in thermodynamic equilibrium,
 b) air can be treated as an ideal gas,
 c) no heat loss from the reactor to the drywell atmosphere,
 d) no steam condensation on the drywell walls,
 e) blowdown takes place at a constant reactor pressure of 500 psia (3.45 MPa) for the duration of interest
 f) no heat loss from drywell through the walls
 g) steam may be treated as an ideal gas so that $u_{\text{Steam}} = u_g + c_{v,\text{Steam}}(T - T_{\text{sat}})$ and $h_{\text{Steam}} = h_g + c_{p,\text{Steam}}(T - T_{\text{sat}})$, where $u_g = 1098$ Btu/lbm (2554 kJ/kg), $h_g = 1180$ Btu/lbm (2744 kJ/kg), and $T_{\text{sat}} = 776$ R (431 K).
 [Ans.: $t = 89$ s, $P_2 = 70$ psia, $T_2 = 475$ F (246 C), $\phi_2 = 12\%$, $m_a = 35,439$ lbm (16,286 kg)].

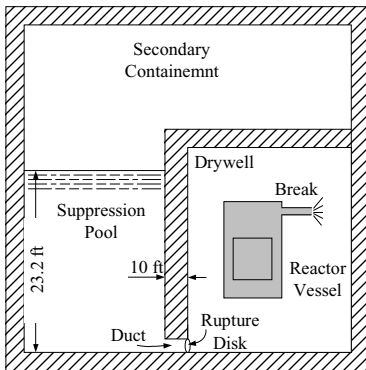


Figure for Problem 20

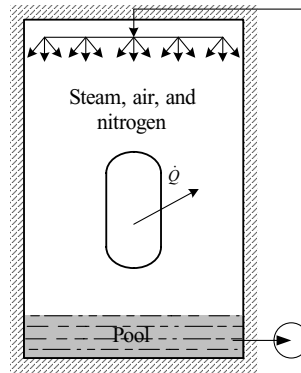


Figure for Problem 21

21. A large containment is filled with steam, air, and nitrogen. The containment also has a pool of water, which is sprayed in the containment atmosphere. Heat is added to the containment at a specified rate. Use the given data to find a) the initial containment pressure and b) the time it takes for the containment pressure to reach the pressure limit of 145 psia (1 MPa).

Total mass of water (water in the pool and steam) $m_w = 3.42\text{E}6$ lbm (1.55E6 kg)

Water-steam quality $x = 0.03$

Air mass $m_a = 0.132\text{E}6$ lbm (6E4 kg)

Nitrogen mass $m_n = 2200$ lbm (998 kg)

Containment initial temperature $T_i = 688$ R (109 C)

Initial relative humidity $\phi_i = 100\%$

Rate of heat addition to containment atmosphere $\dot{Q} = 1.02\text{E}8$ Btu/h (30 MW)

[Ans.: 7.66 h].